③ a) g: Lot auf AB durch C , g: $x + 3y - 3 = 0$

h: Gerade durch A und B , h: $\vec{r} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Schnittpunkt S von g und h: $1 + \mu + 12 + 9\mu - 3 = 0 \rightarrow \mu = -1 \rightarrow S(0/1)$

$\vec{OD} = \vec{OA} - \vec{BS} + \vec{SC} = \begin{pmatrix} -2 \\ -5 \end{pmatrix} - \begin{pmatrix} -1 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \rightarrow \underline{\underline{D(-4/-1)}}$

b) $\vec{AB} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$, $\vec{AD} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$; $\cos\alpha = \dfrac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}||\vec{AD}|} = \dfrac{30}{\sqrt{90}\sqrt{20}} \rightarrow \underline{\underline{\alpha = 45^\circ}}$

c) Trapezfläche A = $2 \cdot$ Dreiecksfläche BCS + Rechtecksfläche

$= 2 \cdot \frac{1}{2}\, \overline{BS} \cdot \overline{SC} + \overline{CD} \cdot \overline{SC} = \sqrt{10} \cdot \sqrt{10} + \sqrt{10} \cdot \sqrt{10} = \underline{20}$

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④ $P(A) = \dfrac{4! \cdot 12!}{16!} = \dfrac{24}{16 \cdot 15 \cdot 14 \cdot 13} = \dfrac{1}{1820} = \underline{\underline{0.055\,\%}}$

$P(B) = 1 - \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 12!}{16!} = \dfrac{265}{364} = \underline{\underline{72.8\%}}$

$P(C) = \dfrac{(16 \cdot 12 \cdot 8 \cdot 4) \cdot (12 \cdot 9 \cdot 6 \cdot 3) \cdot (8 \cdot 6 \cdot 4 \cdot 2) \cdot (4 \cdot 3 \cdot 2 \cdot 1)}{16!} = \underline{\underline{0.526\,\%}}$

$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

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⑤ Flächeninhalt $A = c^2 - \pi r^2$; c: Kantenlänge des Quadrats

Nebenbedingung $U = 4c - 8r + 2\pi r = 100$

$\rightarrow c = 25 + 2r - 0.5\pi r$

einsetzen $\rightarrow A(r) = (25 + 2r - 0.5\pi r)^2 - \pi r^2 = 625 + (100 - 25\pi) \cdot r + (2 - \frac{\pi}{2})^2 \cdot r^2 - \pi r^2$

$A'(r) = 100 - 25\pi + (8 - 4\pi + \frac{\pi^2}{2}) \cdot r - 2\pi r = 100 - 25\pi + (8 - 6\pi + \frac{\pi^2}{2}) \cdot r$

setze $A'(r) = 0 \rightarrow \underline{\underline{r = \dfrac{(100 - 25\pi) \cdot 2}{12\pi - 16 - \pi^2}}}$

$A''(r) = 8 - 6\pi + \frac{\pi^2}{2} < 0 \rightarrow$ Maximum ✓

(ohne Betrachtung der Randwerte, weil A_{max} nicht berechnet werden soll.)

1b) Falls die allgemeine Funktion gemeint ist:

$f(x) = x$

$x_1 = 0$; $x_{2,3} = \pm \sqrt{\frac{(k+1)}{k}}$ $k > -1$

$I_1 = \int_0^{x_2} (x - f(x))\,dx = [-\frac{k}{4}x^4 + \frac{k+1}{2}x^2]_0^{x_1} = (k+1)^2/4k$

$I_2 = -\int_0^1 f(x)\,dx = -[\frac{k}{4}x^4 - \frac{k}{2}x^2]_0^1 = \frac{k}{4}$

$I_1 - I_2 = \dfrac{2k+1}{4k}$ $\dfrac{I_1 - I_2}{I_2} = \dfrac{2k+1}{k^2}$

Passerelle
Sommer 2016
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(1) a) $f'(x) = 3kx^2 - k$, $f''(x) = 6kx$

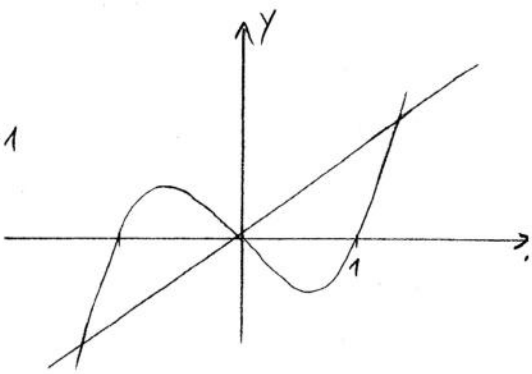
setze $f''(x) = 0$: $6kx = 0 \rightarrow x = 0$

Bedingung $f'(0) = -1 \rightarrow -k = -1 \rightarrow k = 1$

$\rightarrow f(x) = x^3 - x$

Nullstellen : $x = 0$, $x = \pm 1$

$\lim\limits_{x \to +\infty} f(x) = +\infty$, $\lim\limits_{x \to -\infty} f(x) = -\infty$



b) Schnittstellen von f und der 1. Winkelhalbierenden:

$x^3 - x = x \rightarrow x^3 - 2x = 0 \rightarrow x \cdot (x^2 - 2) = 0 \rightarrow x = 0,\ x = \pm\sqrt{2}$

$$I_1 = \int_0^{\sqrt{2}} \left(x - (x^3 - x)\right) dx = \int_0^{\sqrt{2}} (2x - x^3)\, dx = \left[x^2 - \frac{x^4}{4}\right]_0^{\sqrt{2}} = 1 = A_1$$

$$I_2 = \int_0^1 f(x)\, dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1 = -\frac{1}{4} \rightarrow A_2 = 0.25$$

Verhältnis $1 : 3$

allg. Lsg ↷

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(2) a) Nullstellen: $x^2 \cdot (2 - x^2) = 0 \rightarrow \underline{\underline{N(0/0)}}, \underline{\underline{N(\sqrt{2}/0)}}, \underline{\underline{N(-\sqrt{2}/0)}}$

$f'(x) = 4x - 4x^3$, $f''(x) = 4 - 12x^2$, $f'''(x) = -24x$.

setze $f'(x) = 0$: $4x \cdot (1 - x^2) = 0 \rightarrow x = 0$, $x = \pm 1$

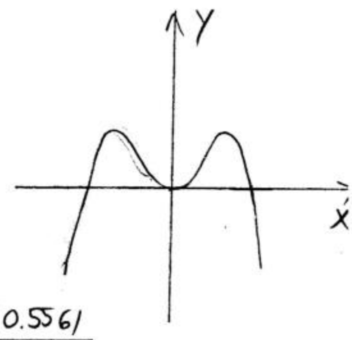
$f''(0) > 0 \rightarrow \underline{\underline{T(0/0)}}$

$f''(1) < 0 \rightarrow \underline{\underline{H(1/1)}}$, $f''(-1) < 0 \rightarrow \underline{\underline{H(-1/1)}}$

setze $f''(x) = 0$: $x^2 = \frac{1}{3} \rightarrow x = \pm\sqrt{1/3}$

$f'''(\pm\sqrt{1/3}) \neq 0 \rightarrow \underline{\underline{W(0.577/0.556)}}, \underline{\underline{W(-0.577/0.556)}}$

Graph achsensymm. zur y-Achse, weil nur gerade Potenzen.



b) $f'(x) = 0 \rightarrow 2ax - 4bx^3 = 0 \rightarrow 2x \cdot (a - 2bx^2) = 0 \rightarrow x = 0,\ x = \pm\sqrt{\frac{a}{2b}}$

$T(0/0)$, $H\left(\pm\sqrt{\frac{a}{2b}} / \frac{a^2}{4b}\right)$

$$f\left(\sqrt{\tfrac{a}{2b}}\right) = a \cdot \frac{a}{2b} - b \cdot \frac{a^2}{4b^2} = \frac{a^2}{2b} - \frac{a^2}{4b} = \frac{a^2}{4b}$$

Höhe $h = 1 \rightarrow$ Seitenlänge $s = \frac{2}{\sqrt{3}}$

Bedingungen: 1) $\frac{a^2}{4b} = 1 \rightarrow a^2 = 4b$

2) $2 \cdot \sqrt{\frac{a}{2b}} = \frac{2}{\sqrt{3}} \rightarrow \frac{a}{2b} = \frac{1}{3} \rightarrow b = \frac{3}{2}a$

$\rightarrow a^2 = 6a \rightarrow (a = 0)$, $\underline{\underline{a = 6}}$, $\underline{\underline{b = 9}}$

$\rightarrow f(x) = 6x^2 - 9x^4$; Nullstellen $x = 0$, $x = \pm\sqrt{2/3}$

$$A = \int_0^{\sqrt{2/3}} f(x)\, dx = \left[2x^3 - \frac{9}{5}x^5\right]_0^{\sqrt{2/3}} = \frac{4}{3} \cdot \sqrt{\frac{2}{3}} - \frac{9}{5} \cdot \frac{4}{9} \cdot \sqrt{\frac{2}{3}} = \frac{8}{15} \cdot \sqrt{\frac{2}{3}} = \underline{\underline{0.4355}}$$