Püss; H10

1. 4 faire Würfel

a) WS(lauter verschiedene Augenzahlen)

$= \frac{|\text{günstige}|}{|\text{mögliche}|} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{6^4} = \underline{\frac{5}{18} = 27{,}8\%}$

b) WS(Augensumme > 20)

$AS = 24:$ (6666) 1 mal
$AS = 23:$ (6665) 4 mal
$AS = 22:$ (6655) 6 mal
(6664) 4 mal
$AS = 21:$ (6663) 4 mal
(6654) 12 mal
(6555) 4 mal

} 35 $\underline{WS(AS > 20) = \frac{35}{6^4} = \frac{35}{1296} = 2{,}7\%}$

c) WS(Xaver gewinnt) = WS(eine "1" oder drei "1")

$(1\bar{1}\bar{1}\bar{1})$ 4 mal $(111\bar{1})$ 4 mal
$1 \cdot 5 \cdot 5 \cdot 5$ $1 \cdot 1 \cdot 1 \cdot 5$
5^3 5

} $4 \cdot 5^3 + 4 \cdot 5 = 520$ $\frac{520}{1296} = \frac{65}{162} = \underline{40\%}$

WS(Yvonne gewinnt) = WS(zwei "1" oder vier "1")

$(11\bar{1}\bar{1})$ 6 mal (1111) 1 mal
$1 \cdot 1 \cdot 5 \cdot 5$ 1
5^2

} $6 \cdot 5^2 + 1 = 151$ $\frac{151}{1296} = \underline{11{,}6\%}$

2. $a \in \mathbb{R}^+$ so, daß $t(x) = \frac{4}{5}x + \frac{9}{5}$ Tangente an $f(x) = \sqrt{a+x^2}$

Tangente: I $t(x) = f(x)$
II $t'(x) = f'(x)$

I. $\frac{4}{5}x + \frac{9}{5} = \sqrt{a+x^2}$

II $\frac{4}{5} = \frac{\cancel{2}x}{\sqrt{a+x^2}} \Rightarrow \sqrt{a+x^2} = \frac{5}{4}x$ in I:

I $\frac{4}{5}x + \frac{9}{5} = \frac{5}{4}x$

$\underline{x = 4}$

in II. $\sqrt{a+4^2} = \frac{5}{4} \cdot 4$

$\underline{\underline{a = 9}}$

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3. Kreiszylinder, $V = \text{konst.}$; $O \to \min$?

$V = \pi r^2 h \qquad O = 2\pi r^2 + 2\pi r h$

$h = \frac{V}{\pi r^2} \longrightarrow O(r) = 2\pi r^2 + 2\pi r \cdot \frac{V}{\pi r^2} \qquad D =]0; \infty[$

$O(r) = 2\pi r^2 + \frac{2V}{r}$

$O'(r) = 4\pi r - \frac{2V}{r^2}$

$O''(r) = 4\pi + \frac{4V}{r^3} > 0$ in D

$O'(r) = 0$

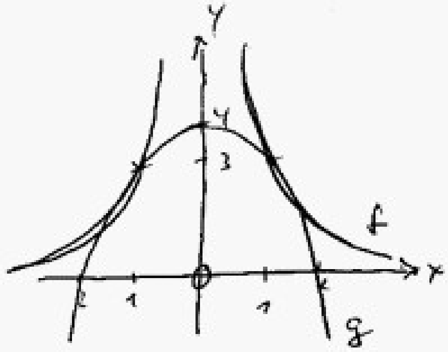
$\underline{r^* = \sqrt[3]{\frac{V}{\pi}}} \qquad O''\left(\sqrt[3]{\frac{V}{\pi}}\right) > 0 \Rightarrow$ Min

~~Ocosse~~ $\lim_{r\to 0} O(r) = \infty$
$\lim_{r\to\infty} O(r) = \infty$ } nach oben unbegrenzt

} Min: $\underline{r = \sqrt[3]{\frac{V}{\pi}}}$

$\underline{O = 4\sqrt[3]{\pi}\sqrt[3]{V^2}}$

4. $f(x) = \frac{3}{x^2}$; $g(x) = a - x^2$



a) $a = 4 \qquad \frac{3}{x^2} = 4 - x^2$

$x^4 - 4x^2 + 3 = 0$

$u^2 - 4u + 3 = 0$

$x_{1/2} = \pm 1 \quad y_{1/2} = 3 \qquad \underline{S_{1/2}(\pm 1 | 3)}$

$x_{3/4} = \pm\sqrt{3} \quad y_{3/4} = 1 \qquad \underline{S_{3/4}(\pm\sqrt{3} | 1)}$

$f'(x) = -\frac{6}{x^3}$; $g'(x) = -2x$

$f'(1) = -6 = \tan\alpha_1 \Rightarrow \alpha_1 = -80{,}5°$
$g'(1) = -2 = \tan\alpha_2 \Rightarrow \alpha_2 = -63{,}4°$ } $\underline{\alpha = 17{,}1°}$ bei $S_{1/2}$

$A = \int_1^{\sqrt{3}} (g(x) - f(x))\,dx = \left[+\frac{3}{x} + 4x - \frac{1}{3}x^3\right]_1^{\sqrt{3}} = \underline{4\sqrt{3} - \frac{20}{3}}$

b) I. $\frac{3}{x^2} = a - x^2$

II. $-\frac{6}{x^3} = -2x \Rightarrow x = \pm\sqrt[4]{3}$ in I: $\underline{a = 2\sqrt{3}}$

$y = \sqrt{3}$

$m = g'(\sqrt[4]{3}) = -2\sqrt[4]{3}$; $B(\sqrt[4]{3} | \sqrt{3})$

$t: y = m(x - x_0) + y_0$

$\underline{t: y = -2\sqrt[4]{3}(x - \sqrt[4]{3}) + \sqrt{3}} = \underline{-2\sqrt[4]{3}\,x + 3\sqrt{3}}$

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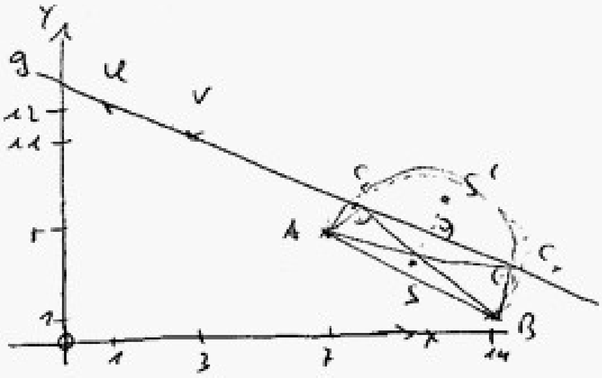
5. g durch $U(1|12)$ $V(3|11)$

$\triangle ABC$: $A(7|5)$ $B(14|1)$

$C \in g$ so $AC \perp CB$



g: $m = -\frac{1}{2}$

$y = -\frac{1}{2}(x-1)+12 = -\frac{1}{2}x + \frac{25}{2}$

$\perp C$: $\vec{AC} \cdot \vec{CB} = 0$

$\begin{pmatrix} x-7 \\ y-5 \end{pmatrix}\begin{pmatrix} 14-x \\ 1-y \end{pmatrix} = 0$

$(x-7)(14-x) + (-\frac{1}{2}x + \frac{25}{2} - 5)(1 + \frac{1}{2}x - \frac{25}{2}) = 0$

$\underline{x_1 = 11}$ $\underline{y_1 = 7}$ $\underline{C_1(11|7)}$

$x_2 = 67/5$

Mit dem Dreieck wird auch sein Schwerpunkt gespiegelt.

$\vec{r}_S = \frac{\vec{r}_A + \vec{r}_B + \vec{r}_C}{3} = \begin{pmatrix} 32/3 \\ 13/3 \end{pmatrix}$

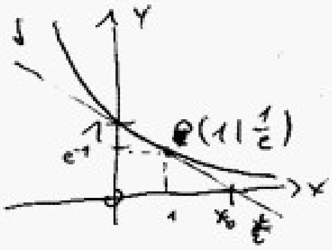
Abstand S zu g: HNF(g): $\frac{x+2y-25}{\sqrt{5}} = 0$

$d(S;g) = \frac{17}{3\sqrt{5}}$

$\vec{n}_g = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\vec{e}_{n_g} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\vec{SS'} = 2 \cdot \frac{17}{3\sqrt{5}} \cdot \vec{e}_{n_g} = 2 \cdot \frac{17}{3\sqrt{5}} \cdot \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{34}{15}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\underline{\vec{r}_{S'}} = \vec{r}_S + \vec{SS'} = \begin{pmatrix} 32/3 \\ 13/3 \end{pmatrix} + \frac{34}{15}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \underline{\begin{pmatrix} 194/15 \\ 133/15 \end{pmatrix}}$

6 a) $f(x) = e^{-x}$



$f'(x) = -e^{-x}$

$f'(1) = -e^{-1} = m$

t: $y = -\frac{1}{e}(x-1) + \frac{1}{e}$

$\underline{y = -\frac{1}{e}x + \frac{2}{e}}$

b) $g(x) = e^{kx}$; $k < 0$ $g(1) = e^k$

$g'(x) = ke^{kx}$ $g'(1) = ke^k$

$P(1|e^k)$ t: $y = ke^k(x-1) + e^k$

t: $\underline{y = ke^k x + e^k - ke^k}$

NST: $0 = ke^k x + e^k - ke^k$

$\underline{x_0 = \frac{k-1}{k}}$

$F = \int_0^{x_0} g(x)\,dx = \left[\frac{1}{k}e^{kx}\right]_0^{x_0}$

$\underline{F(k)} = \frac{1}{k}e^{k-1} - \frac{1}{k} = \underline{\frac{1}{k}(e^{k-1} - 1)}$

$\underline{\lim_{k \to 0}} \underbrace{\frac{1}{k}}_{\to -\infty}\underbrace{(e^{k-1} - 1)}_{\to e^{-1}-1<0} \underline{= \infty}$

$\underline{\lim_{k \to -\infty}} \underbrace{\frac{1}{k}}_{\to 0}\underbrace{(e^{k-1} - 1)}_{\to -1} \underline{= 0}$