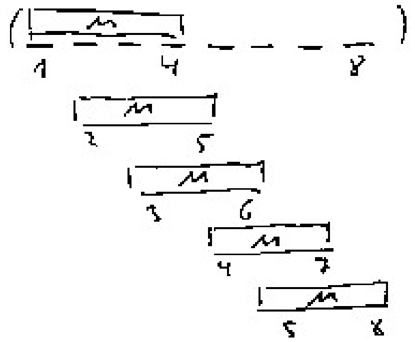
Passerelle, Sommer 2009

1. a)



8! mögliche Anordnungen der Jungen und Mädchen insgesamt.

5 mögliche Positionen für den 4er Block Mädchen

4! mögliche Anordnungen der Mädchen innerhalb eines 4er Blocks (dito für die Jungen)

$$WS = \frac{5\ 4!\ 4!}{8!} = \underline{\frac{1}{14}}$$

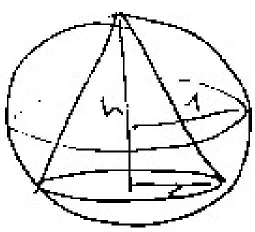
b) WS(nicht abwechslungsweise K, M)

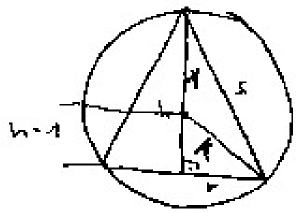
$= 1 - \underbrace{WS(\text{abwechslungsweise } K, M)}$

$$\left.\begin{matrix} KMKMKMKM \\ MKMKMKMK \end{matrix}\right\} 2$$

$$= 1 - 2 \cdot \frac{4}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \underline{\frac{34}{35}}$$

2.





$$1^2 = (h-1)^2 + r^2$$
$$1 = h^2 - 2h + 1 + r^2$$
$$\underline{r^2 = 2h - h^2}$$
$$s^2 = r^2 + h^2 = 2h - h^2 + h^2 = 2h$$
$$s = \sqrt{2h}$$

$$O = r^2\pi + \pi r s$$

$$= r^2\pi + \pi r \sqrt{2}\cdot\sqrt{h} = (2h - h^2)\pi + \pi\sqrt{2}\sqrt{2h-h^2}\sqrt{h}$$

$$O = \pi\left(2h - h^2 + \sqrt{4h^2 - 2h^3}\right)$$

$$O' = \pi\left(2 - 2h + \frac{1}{2\sqrt{4h^2-2h^3}}(8h - 6h^2)\right) = \pi\left(2 - 2h + \frac{4-3h}{\sqrt{4-2h}}\right) = 0$$

$$(2-2h)^2(4-2h) = (4-3h)^2$$

$$h(\underbrace{-8h^2 + 23h - 16}) = 0$$

$h = 0 \qquad h = \frac{23 \pm \sqrt{17}}{16}$

$$\underline{h_{max} = \frac{23 + \sqrt{17}}{16}} = \underline{1.7}$$

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3. $h(x) = \sin 2x + a\cos x \qquad \mathbb{D} = [0; 2\pi]$

in $x = \frac{\pi}{6}$ Ext.

$h'(x) = 2\cos 2x - a\sin x$

$h'(\frac{\pi}{6}) = 0$

$2\cos(\frac{\pi}{3}) - a\sin(\frac{\pi}{6}) = 0$

$1 - a\frac{1}{2} = 0$

$\underline{a = 2}$

$h'(x) = 2\cos 2x - 2\sin x = 0 \qquad\qquad h(x) = \sin 2x + 2\cos x$

$1 - 2\sin^2 x - \sin x = 0$

$2\sin^2 x + \sin x - 1 = 0$

$\sin x_{1/2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

$\{\sin x_1 = -1\} \qquad\qquad \sin x_2 = \frac{1}{2}$

$x_1 = \frac{3\pi}{2} \qquad\qquad x_2 = \frac{\pi}{6} \quad y_2 = \frac{3}{2}\sqrt{3}$

$y_1 = 0 \qquad\qquad x_3 = \frac{5}{6}\pi \quad y_3 = -\frac{3}{2}\sqrt{3}$

$h''(x) = -4\sin 2x + 2\cos x$

$h''(\frac{3}{2}\pi) = 0$

$h''(\frac{\pi}{6}) < 0 \rightarrow Max(\frac{\pi}{6} \mid \frac{3}{2}\sqrt{3})$

$h''(\frac{5}{6}\pi) > 0 \Rightarrow Min(\frac{5}{6}\pi \mid -\frac{3}{2}\sqrt{3})$

VZT

x	π	$\frac{3}{2}\pi$	2π
$h'(x)$	+	0	+
	$\nearrow$	$\rightarrow$	$\nearrow$

Terrassenpunkt $(\frac{3}{2}\pi \mid 0)$, kein Extremwert

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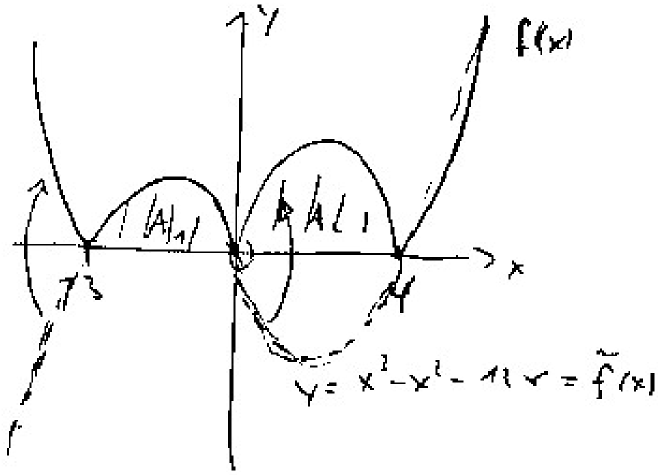
4. $f(x) = |x^3 - x^2 - 12x|$

$x(x^2 - x - 12) = 0$

$x = 0 \quad x = 4 \quad x = -3$



$$A = A_1 + A_2 = \int_{-3}^{0} \tilde{f}(x)\,dx + \int_{0}^{4} -\tilde{f}(x)\,dx$$

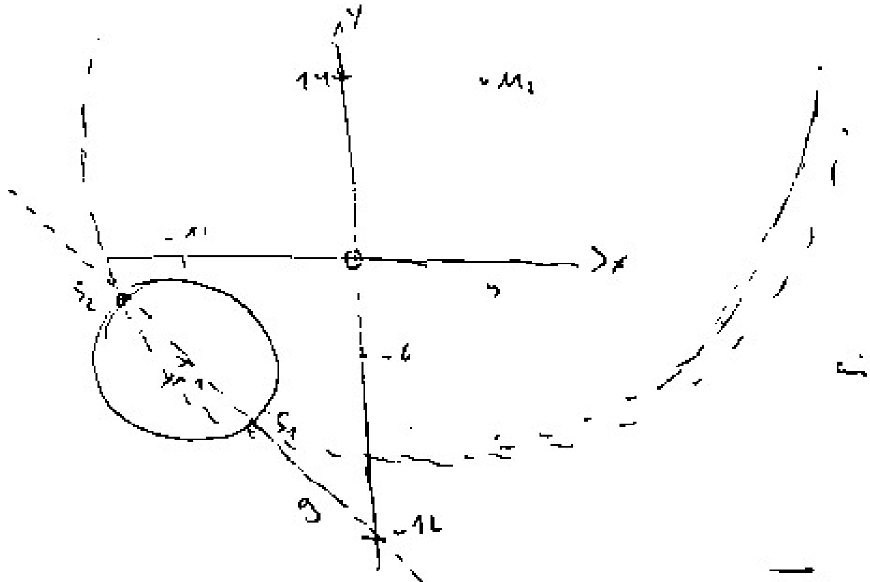
$$= \left[\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - 6x^2\right]_{-3}^{0} - \left[\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 - 6x^2\right]_{0}^{4}$$

$$= \frac{99}{4} - \left(-\frac{160}{3}\right)$$

$$= \underline{\frac{937}{12}}$$

5. $k_1: \; x^2 + y^2 + 22x + 12y + 132 = 0 \rightarrow \text{I}.\ (x+11)^2 + (y+6)^2 = 5^2 \qquad M_1(-11|-6), \; R_1 = 5$

$k_2: \; x^2 + y^2 - 18x - 28y - 348 = 0 \rightarrow \text{II}\ (x-9)^2 + (y-14)^2 = 25^2 \qquad M_2(9|14), \; R_2 = 25$



$k_1 \cap k_2$:

$\text{I} - \text{II}: \quad x + y + 12 = 0$

$g: \; y = -x - 12$

in I: $\quad x^2 + 17x + 66 = 0$

$(x+6)(x+11) = 0$

$x_1 = -6 \;;\; y_1 = -6$

$x_2 = -11 \;,\; y_2 = -1$

$\overrightarrow{S_1S_2} = \begin{pmatrix} -5 \\ +5 \end{pmatrix} \qquad \underline{|\overrightarrow{S_1S_2}| = 5\sqrt{2}}$

$\underline{R_2'} = R_1 + |\overrightarrow{M_1M_2}| = 5 + \left|\begin{pmatrix} 20 \\ 20 \end{pmatrix}\right| = \underline{5 + 20\sqrt{2}}$

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6. $\vec{a} = \begin{pmatrix} x \\ 2 \\ 5 \end{pmatrix}$; $\vec{b} = \begin{pmatrix} 1 \\ y \\ -2 \end{pmatrix}$

a) $x = 1, \ y = 2$

$$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{a\cdot b} = \frac{\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}{\sqrt{30}\sqrt{9}} = \frac{1+4-10}{3\sqrt{30}} = \frac{-5}{3\sqrt{30}}$$

$$\underline{\alpha = 107{,}7^\circ}$$

b) $\vec{a}\cdot\vec{b} = 0 \iff \vec{a} \perp \vec{b}$

$x + 2y - 10 = 0$

$\underline{x = 10 - 2y}$

c) $\vec{a}\cdot\vec{b} = 0 \quad \wedge \quad |\vec{a}| = |\vec{b}|$

$x = 10 - 2y \quad \wedge \quad x^2 + 4 + 25 = 1 + y^2 + 4$

$\quad '' \quad \wedge \quad x^2 + 24 = y^2$

$(10 - 2y)^2 + 24 = y^2$

$$y = \frac{10 \pm 2\sqrt{7}}{3}$$

$$x = \frac{10 \mp 4\sqrt{7}}{3}$$

d) $\vec{a}\cdot\vec{b} = 0 \quad \wedge \quad |\vec{a}| = 2|\vec{b}|$

$x = 10 - 2y \quad x^2 + 29 = 4(y^2 + 5)$

$x^2 + 9 = 4y^2$

$(10 - 2y)^2 + 9 = 4y^2$

$$y = \frac{109}{40}$$

$$x = \frac{91}{20}$$