Lösungen "Passerelle Herbst Basel 05"

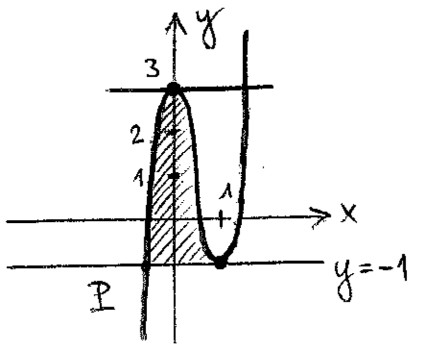
1) $f(x) = 8x^3 - 12x^2 + 3$

$f'(x) = 24x^2 - 24x = 24x(x-1)$ $\Big\}$ $f'(x)=0$ $\Big\{$ $x_1 = 0 \quad y_1 = 3$, $x_2 = 1 \quad y_2 = -1$

$f''(0) = -24$, $f''(1) = 24$ $\Big\}$ $\underline{\underline{H(0/3)}}$, $\underline{\underline{T(1/-1)}}$

$f''(x) = 48x - 24$

Skizze Bild:



Fläche: $f(x) - (-1)$

$$A = \int_{-\frac{1}{2}}^{1} (8x^3 - 12x^2 + 4)\,dx = \left[2x^4 - 4x^3 + 4x\right]_{-\frac{1}{2}}^{1}$$

$$\underline{\underline{A = \frac{27}{8}}}$$

P: $8x^3 - 12x^2 + 3 = -1$

Polynomdivision $(x_1 = 1)$

$8x^3 - 12x^2 + 4 : x - 1 = 8x^2 - 4x - 4$

$\underline{-\ 8x^2}$

$-4x^2 + 4$

$\underline{+4x}$

$-4x + 4$

$2x^2 - x - 1 = 0$

$x_2 = 1$

$x_3 = -\frac{1}{2}$

$P(-\frac{1}{2}/-1)$

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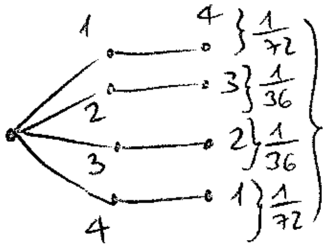
2)

a) $P = \left(\frac{1}{4}\right)^3 + 4 \cdot \left(\frac{1}{6}\right)^3 + \left(\frac{1}{12}\right)^3 = \underline{\underline{\frac{5}{144}}}$

b)



$P = \frac{2/72}{6/72} = \underline{\underline{\frac{1}{3}}}$

c) Gegenwahrscheinlichkeit: (keine 6) $\quad \overline{P} = \left(\frac{3}{4}\right)^n \leq 0{,}005$

$n \log(0{,}75) \leq \log(0{,}005)$

$n \geq \frac{\log(0{,}005)}{\log(0{,}75)}$ $\Big\{$ neg. Log-werte

$(\ \underline{n \geq 18{,}4}\ )$

Vanzetti muss $\underline{\underline{19 \text{ mal}}}$ würfeln

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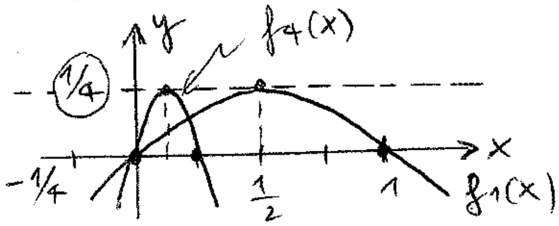
3) $f_k(x) = kx - k^2x^2$

a) $f_1(x) = x - x^2$

$f_4(x) = 4x - 16x^2$

$\Big\{$ $k > 0$ reell



b) NS: $kx(1 - kx) = 0$ $\Big\{$ $\underline{\underline{x_1 = 0}}$, $\underline{\underline{x_2 = \frac{1}{k}}}$ $\Big\}$

relative Extrema: $f_k'(x) = k - 2k^2x = k(1 - 2kx)$

$f_k'(x) = 0$ $\Big\{$ $x = \frac{1}{2k}$, $y = \frac{1}{4}$ $\Big\}$ $\underline{\underline{H\left(\frac{1}{2k}/\frac{1}{4}\right)}}$

$(f_k''(x) = -2k^2)$

c) laut b) $\underline{\underline{y_H = \frac{1}{4}}}$

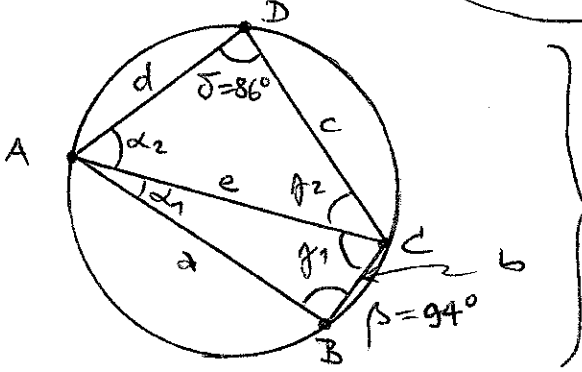
d) $\frac{1}{4} = \int_0^{1/k} (kx - k^2x^2)\,dx = \left[\frac{kx^2}{2} - \frac{k^2x^3}{3}\right]_0^{1/k} = \left(\frac{1}{2k} - \frac{1}{3k}\right) - 0$

$\frac{1}{4} = \frac{1}{6k}$ $\quad | \quad$ $\underline{k = \frac{2}{3}}$

Lösungen "Pss. H. Basel 05" ; Folge

nicht im Stoffplan ..

4) Bemerkung: Diese Aufgabe kann nur sinnvoll mit dem Sinussatz gelöst werden (Formelbuch ~~(Klaus)~~ S. 28 "Berechnung des allg. Dreiecks")
Ferner ist unklar, was $a = |AB| = 5,$ bedeuten soll. Meine Annahme $\underline{a = 5{,}0}$



Sinussatz: $\sin\gamma_1 = \frac{a \sin\beta}{e}$ $\quad \underline{\gamma_1 = 67{,}47^\circ}$, $\underline{\alpha_1 = 18{,}53^\circ}$

$b = e\left(\frac{\sin\alpha_1}{\sin\beta}\right)$ $\quad \underline{\underline{b = 1{,}72}}$

$\sin\alpha_2 = \left(\frac{c}{e}\right)\sin\delta$ $\quad \alpha_2 = 52{,}59^\circ$

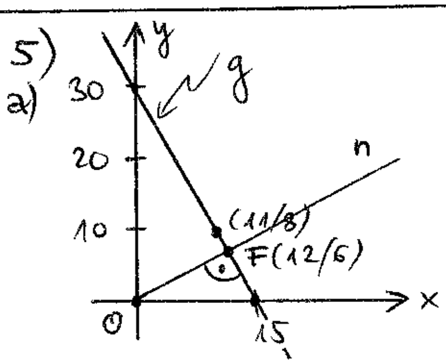
$\underline{\underline{\alpha = \alpha_1 + \alpha_2 = 71{,}12^\circ}}$

$\underline{\underline{d}} = e\left(\frac{\sin\gamma_2}{\sin\delta}\right) = \underline{\underline{3{,}58}}$

$(\gamma_2 = 41{,}41^\circ)$

(gegenüberliegende Winkel sind im Sehnenviereck supplementär
also: $\beta + \delta = 180^\circ \longrightarrow \delta = 86^\circ$
Formelbuch (Klaus) S. 22)

5)
a)



1) $g: \begin{pmatrix} 11 \\ 8 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $n: \begin{pmatrix} 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad F = g \cap n$ $\left\{ \begin{aligned} 11 + t &= 2s \\ 8 - 2t &= s \quad \vert \cdot(-2) \\ \hline -5 + 5t &= 0 \end{aligned} \right\} t = 1$

Somit: $\underline{\underline{F(12/6)}}$

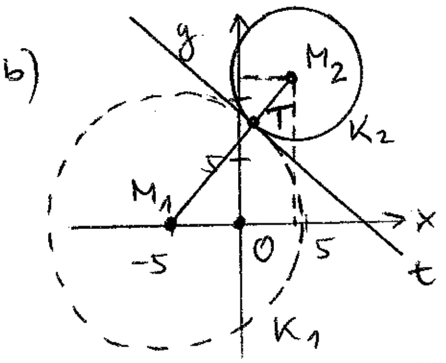
Abstand $\underline{\underline{a}} = |\overline{OF}| = \sqrt{12^2 + 6^2} = \underline{\underline{\sqrt{180}}}$ $(\cong 13{,}4)$

2) $\vec{f} = \begin{pmatrix} 12 \\ 6 \end{pmatrix} = \lambda\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $\quad \begin{aligned} 12 &= 5\lambda + 6\mu \\ 6 &= -\lambda + 2\mu \quad \vert \cdot 5 \end{aligned}$

$\begin{aligned} 42 &= 16\mu \\ \lambda &= 2\mu - 6 = -\frac{3}{4} \end{aligned}$ $\quad \mu = \frac{21}{8}$

$\underline{\underline{\vec{f} = -\frac{3}{4}\vec{a} + \frac{21}{8}\vec{b}}}$

b)

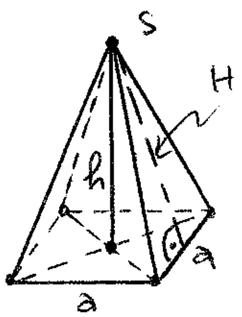


$\frac{A_1}{A_2} = 4$; $\frac{r_1}{r_2} = 2$; $\overrightarrow{M_1T} = \frac{2}{3}\overrightarrow{M_1M_2} = \frac{2}{3}\begin{pmatrix} 9 \\ 12 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$\underline{\overrightarrow{OT} = \overrightarrow{OM_1} + \overrightarrow{M_1T}} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ 8 \end{pmatrix}}$ $\quad \underline{\underline{T(1/8)}}$

$\underline{t:\ \vec{r}_x = \begin{pmatrix} 1 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}}$ $\quad$ ($\overrightarrow{M_1M_2} = 3\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ ⊥ dazu Tangente $\to \begin{pmatrix} 4 \\ -3 \end{pmatrix}$)

6)



$O = 200$
V soll max. werden

$V = a^2 \cdot \frac{h}{3}$

$O = a^2 + 4\left(\frac{1}{2} a H\right) = 200$; $\underline{H^2 = h^2 + \left(\frac{a}{2}\right)^2}$

$h^2 = H^2 - \frac{a^2}{4}$

$H = \frac{200 - a^2}{2a}$ $\quad h^2 = \left(\frac{200 - a^2}{2a}\right)^2 - \frac{a^2}{4} = \frac{-400a^2 + 40'000}{4a^2}$

$\underline{V = \frac{1}{3} a^2 \frac{\sqrt{40'000 - 400a^2}}{2a}}$

$\underline{= \frac{10a}{3}\sqrt{100 - a^2}}$

$V'(a) = \frac{10}{3}\sqrt{\dots} + \frac{10a}{3}\left(\frac{-2a}{2\sqrt{\dots}}\right)$

$V'(a) = 0$ $\quad \underline{\frac{10}{3}(100 - a^2) - \frac{10a^2}{3} = 0}$

$\underline{\underline{a = \sqrt{50}}}$ $\quad a^2 = 50$ $\quad \frac{1000}{3} = \frac{20a^2}{3}$