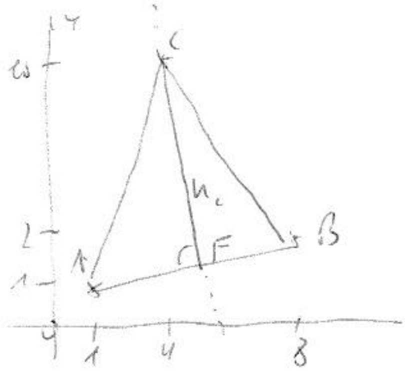
Pass F16

1. a) A(1|1) B(8|2) C(4|10)

$\vec{AB} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

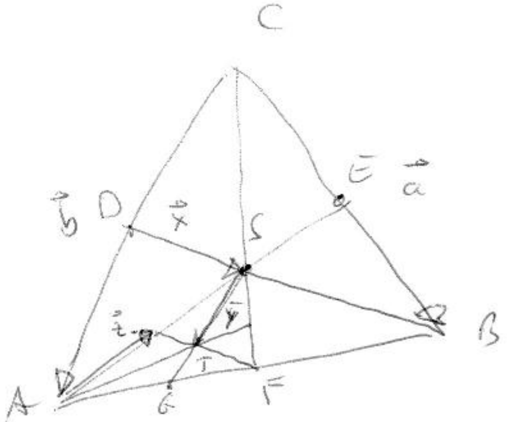


$$\left.\begin{aligned} h_c:\ \vec{x} &= \begin{pmatrix} 4 \\ 10 \end{pmatrix} + t\begin{pmatrix} 1 \\ -7 \end{pmatrix} \\ AB:\ \vec{x} &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + s\begin{pmatrix} 7 \\ 1 \end{pmatrix} \end{aligned}\right\} = \quad s = \frac{3}{5}$$

$t = 6/5$

$\vec{CF} = \frac{6}{5}\begin{pmatrix} 1 \\ -7 \end{pmatrix}$ $\quad h_c = |\vec{CF}| = \frac{6}{5}\sqrt{50} = \underline{6\sqrt{2}}$

b)



S, T: Schwerpunkte im Schnitt der Seitenhalbierenden und diese teilen sich $\underline{2:1}$

$\underline{\vec{z}} = \frac{1}{2}\vec{AS} = \frac{1}{2}\cdot\frac{2}{3}\vec{AE} = \frac{1}{3}\cdot\left(-\vec{b} + \frac{1}{2}\vec{a}\right) = \underline{\frac{1}{6}\vec{a} - \frac{1}{3}\vec{b}}$

$\underline{\vec{x}} = \frac{1}{3}\vec{DB} = \frac{1}{3}\left(-\frac{1}{2}\vec{b} + \vec{a}\right) = \underline{\frac{1}{3}\vec{a} - \frac{1}{6}\vec{b}}$

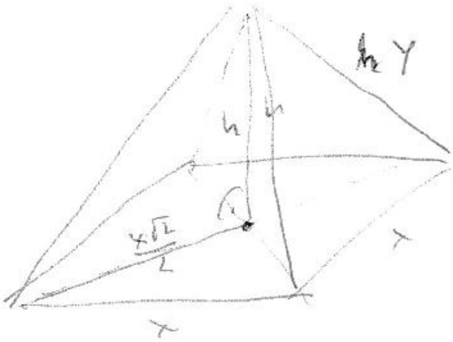
$\underline{\vec{y}} = \frac{2}{3}\vec{SG} = \frac{2}{3}\left(-\vec{x} + \frac{1}{2}\vec{b} + \frac{1}{4}\vec{AB}\right)$

$= \frac{2}{3}\left(-\vec{x} + \frac{1}{2}\vec{b} + \frac{1}{4}\left(-\vec{b} + \vec{a}\right)\right)$

$= \frac{2}{3}\left(-\frac{1}{3}\vec{a} + \frac{1}{6}\vec{b} + \frac{1}{2}\vec{b} - \frac{1}{4}\vec{b} + \frac{1}{4}\vec{a}\right)$

$= \underline{-\frac{1}{18}\vec{a} + \frac{5}{18}\vec{b}}$

2. [Poss F16



$$4y + 4x = 40$$
$$y + x = 10$$
$$y = 10 - x$$

a) $y^2 - h^2 = \left(\frac{x\sqrt{2}}{2}\right)^2 = \frac{x^2}{2}$

$(10-x)^2 - h^2 = \frac{x^2}{2}$

$x^2 - 40x - 2h^2 + 200 = 0$

$\underline{h = \sqrt{\frac{1}{2}(x^2 - 40x + 200)}}$

$\underline{y = 10 - x}$

$\underline{V} = \frac{1}{3}x^2 \cdot h = \underline{\frac{1}{3}x^2\sqrt{\frac{1}{2}(x^2-40x+200)}}$

b) $V = \frac{1}{3\sqrt{2}}\sqrt{x^4(x^2-40x+200)}$

$\sqrt{\ }$ ist streng monoton steigend, also ist $\sqrt{f(x)}$ max, wo $f(x)$ max.

$\mathbb{D} = [0, 20 \mp 10\sqrt{2}]$
5,86

$f(x) = x^6 - 40x^5 + 200x^4$

$f'(x) = 6x^5 - 200x^4 + 800x^3$

$f''(x) = 30x^4 - 800x^3 + 2400x^2$

$f'(x) = 0$

$\underline{x^3}(6x^2 - 200x + 800) = 0$

$x = 0 \in \mathbb{D}$ $\quad x = \frac{50 \pm 10\sqrt{13}}{3}$
$(-) \in \mathbb{D}$

$\underline{\text{Randmin}}$
$\underline{V=0}$

$f''\left(\frac{50 - 10\sqrt{13}}{3}\right) < 0 \Rightarrow \underline{\underline{\text{Max}}}$ bei $\underline{\underline{\frac{1}{3}(50 - 10\sqrt{13})}} \approx$
4,65

$\underline{V(20 - 10\sqrt{2}) = 0 \text{ Randmin}}$

3 $2 \times 3\,cm$
$2 \times 4\,cm$
$2 \times 5\,cm$

PossF16

a) Zmz

(1) $P(\text{gleichseitig}) = 3 \cdot \left(\frac{1}{3}\right)^3 = \frac{1}{9}$

(2) $P(\text{...}) = P(3,4,5) \cdot 6 = 6 \cdot \left(\frac{1}{3}\right)^3 = \frac{2}{9}$

(3) $P(\text{gleichsch.}) = 6\,P(3,3,\overset{4}{5}) + 6 \cdot P(4,4,\overset{5}{3}) + 6P(5,5,\overset{3}{4})$
$= 6 \cdot \left(\frac{1}{3}\right)^3 \cdot 3 = \frac{2}{3}$

b) Z.o.Z.

(1) $0 \quad \left(3 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{0}{4}\right)$

(2) $6 \cdot \frac{2}{6} \cdot \frac{2}{5} \cdot \frac{2}{4} = \frac{2}{5}$

(3) $18 \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{2}{4} = \frac{3}{5}$

c) $P(\text{höchst. einmal in 10 Wdhl. recht.})$

$= P(0\text{ mal}) + P(\text{einmal})$

$= \left(\frac{3}{5}\right)^{10} + \left(\frac{2}{5}\right)^{1}\left(\frac{3}{5}\right)^{9} \cdot 10$

$= 4{,}64\%$

PassF16

4. $f(x) = 2x - \frac{x^3}{2} = \frac{1}{2}x(4-x^2) = \frac{1}{2}x(2-x)(2+x)$

NST: $x = 0;\ \pm 2$

a)

$f'(x) = 2 - \frac{3}{2}x^2$

$f'(0) = 2 \xrightarrow{\perp} -\frac{1}{2} = g'(0)$

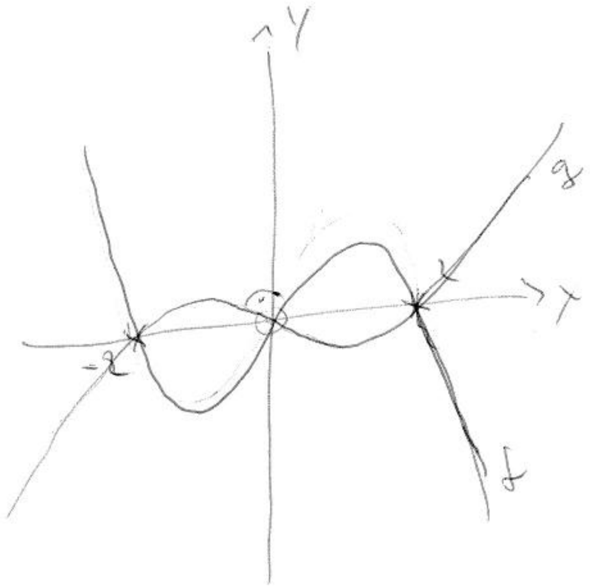
$g(x) = a \cdot x(x-2)(x+2) = a \cdot (x^3 - 4x)$

$g'(x) = a(3x^2 - 4)$

$g'(0) = -4a \stackrel{!}{=} -\frac{1}{2}$

$\underline{a = \frac{1}{8}}$

$\underline{g(x) = \frac{1}{8}(x^3 - 4x)}$



b) Beide sind P.S. da nur ungerade EXP.

$\underline{A} = 2 \cdot \int_0^2 (f-g)\,dx = -\frac{5}{4}\int_0^2 (x^3 - 4x)\,dx$

$= -\frac{5}{4}\left[\frac{1}{4}x^4 - 2x^2\right]_0^2 = \underline{5}$

c) $f'(2) = -4 = \tan\alpha_1 \Rightarrow \alpha_1 = -75{,}96°$
$g'(2) = 1 = \tan\alpha_2 \Rightarrow \alpha_2 = 45°$ $\Big\}$ $\underline{\alpha = 120{,}96°}$

Pass F16

5. $f(x)=\dfrac{x^2+ax+b}{x^2}$; $x>0$

a) $W(2|0)$ WP waag. A.

$f(x) = 1 + ax^{-1} + bx^{-2}$

$f'(x) = -ax^{-2} - 2bx^{-3}$

$f''(x) = 2ax^{-3} + 6bx^{-4}$

I. $f(2)=0$

II. $f''(2)=0$

I $1+\frac{a}{2}+\frac{b}{4}=0$

II. $\frac{a}{4}+\frac{3}{8}b=0$

$\underline{a=-3}$

$\underline{b=2}$

$f(x) = \dfrac{x^2-3x+2}{x^2} = 1-3x^{-1}+2x^{-2}$

$f'(x) = 3x^{-2}-4x^{-3}$

$f'' = -6x^{-3}+12x^{-4}$

b) $f(x)=0$

$(x-2)(x-1)=0$

$\underline{x=2\ ;\ x=1}$

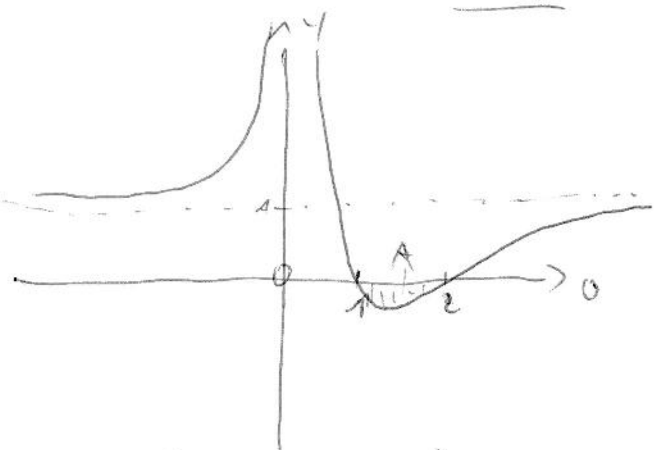
$f'(x)=0$

$\frac{3}{x^2}=\frac{4}{x^3}$

$\underline{x=\frac{4}{3}}$

$\underline{y=-\frac{1}{8}}$

$f''(\frac{4}{3})>0 \Rightarrow \underline{Min(\frac{4}{3}|-\frac{1}{8})}$



c) $A = -\int_1^2 f(x)\,dx = -\int_1^2 (1-3x^{-1}+2x^{-2})\,dx = -\left[x-3\ln x-\frac{2}{x}\right]_1^2$

$\underline{A = 2-3\ln 2}$