Pass IIa
F15

1. $f(x) = 3x^2 - 7x + 8 \qquad g(x) = -x^2 + kx - 8 \qquad k > 0$

I $f(x) = g(x)$
II $f'(x) = g'(x)$

I $3x^2 - 7x + 8 = -x^2 + kx - 8 \longrightarrow 4x^2 \mp (k+7)x + 16 = 0$

Berührung: doppelte Lösung $\Rightarrow D = 0$

II $6x - 7 = -2x + k$

$x = \frac{1}{8}(k+7)$

in I: ... aufwändig

$D = (k+7)^2 - 4 \cdot 4 \cdot 16 = 0$

$k_{1,2} = -7 \pm 16$

$k_1 = 9$

$(k_2 = -23) \notin \mathbb{R}$

Berührung: $x = \frac{1}{8}(k+7) = 2$ $\qquad$ B(2|6)

$y = 6$

NST: $f(x) = 0$

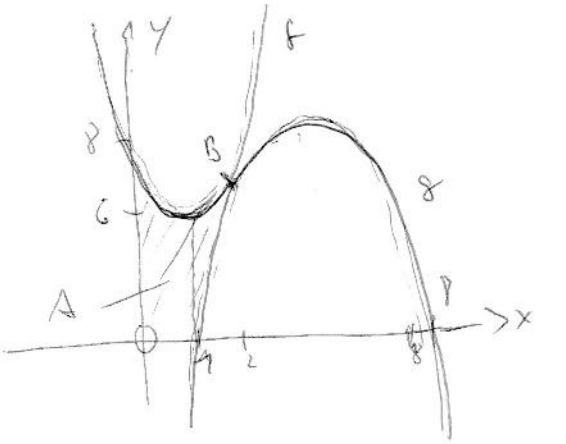
$49 - 4 \cdot 3 \cdot 8 < 0$

keine NST

$g(x) = -x^2 + 9x - 8 = 0$

$x_1 = 1$

$x_2 = 8$



$$A = \int_0^1 f(x)\,dx + \int_1^2 (f-g)\,dx = \int_0^2 f(x)\,dx - \int_1^2 g(x)\,dx$$

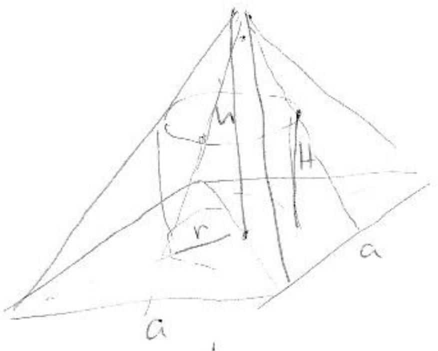
$$= \left[x^3 - \frac{7}{2}x^2 + 8x\right]_0^2 - \left[-\frac{1}{3}x^3 + \frac{9}{2}x^2 - 8x\right]_1^2$$

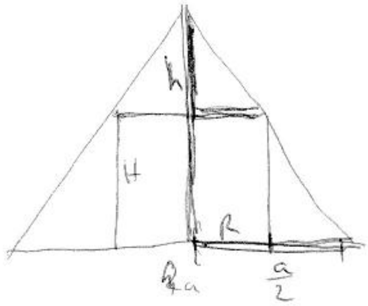
$$= \quad 10 \quad - \quad \frac{19}{6}$$

$$\underline{A = \frac{41}{6}}$$

2.

Pass Ma F15





$$\frac{h}{a/2} = \frac{h-H}{R}$$

$$-\frac{2h}{a}R + h = H$$

$$V_Z = R^2\pi \cdot H = \pi R^2\left(h - \frac{2h}{a}R\right)$$

$$V_Z(R) = \pi h R^2 - \pi\frac{2h}{a}R^3 \rightarrow \max \qquad R \in \left[0; \frac{a}{2}\right]$$

$$V_Z'(R) = 2\pi h R - 6\pi\frac{h}{a}R^2 = 0$$

$$R\underbrace{\left(2h - 6\frac{h}{a}R\right)}_{R = \frac{1}{3}a} = 0$$

$R = 0$ Rand $\qquad R = \frac{1}{3}a$

$$V''(R) = 2\pi h - 12\pi\frac{h}{a}R$$

$$V''\left(\tfrac{1}{3}a\right) = 2\pi h - 12\pi\frac{h}{a}\cdot\frac{1}{3}a = 2\pi h - 4\pi h < 0 \Rightarrow \text{Max bei } R = \frac{1}{3}a$$

Ränder $\left.\begin{array}{l} V(0) = 0 \\ V(\frac{a}{2}) = 0 \end{array}\right\}$ Min

$$V_{max} = \pi h\cdot\left(\tfrac{1}{3}a\right)^2 - \pi\frac{2h}{a}\left(\tfrac{1}{3}a\right)^3$$

$$V_{max} = \frac{1}{27}\pi h a^2$$

$$\frac{V_{max}}{V_{Py}} = \frac{1}{9}$$

PassMa
F15

3. k_1 $M_1(8|0)$ $r_1 = 5$

k_2 $M_2(8|4)$ $r_2 = 3$

I $(x-8)^2 + y^2 = 25$

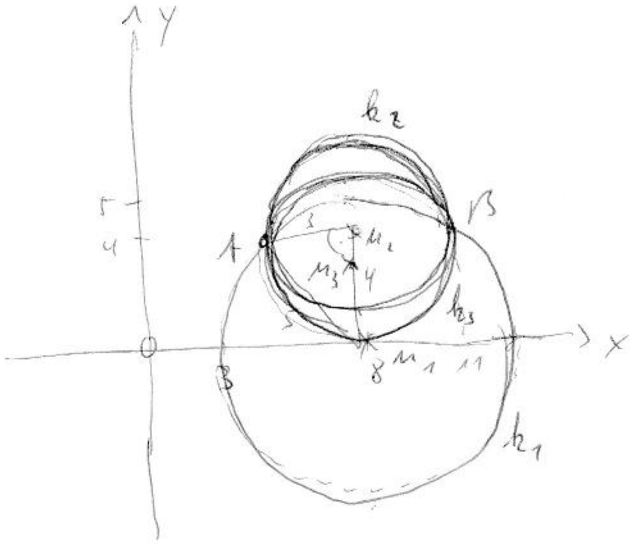
II $(x-8)^2 + (y-4)^2 = 9$

$x_1 = 5$; $y_1 = 4$

$x_2 = 11$; $y_2 = 4$

$A(5|4)$

$B(11|4)$



$k_3(M_1; A; B)$

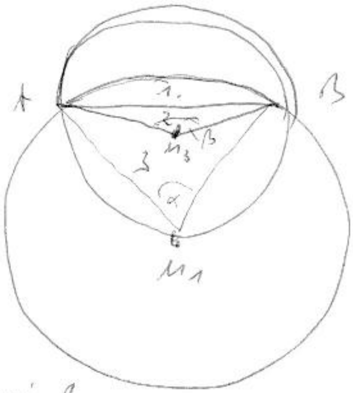
M_3 : $x = 8$ wegen Symmetrie $A|B$

M_3 auf Mittelsenkrechte M_1B: $y = -\frac{3}{4}\left(x - \frac{19}{2}\right) + 2$

$x = 8$: $y = -\frac{3}{4}\left(8 - \frac{19}{2}\right) + 2$

$y = \frac{25}{8}$

k_3: $(x-8)^2 + \left(y - \frac{25}{8}\right)^2 = \left(\frac{25}{8}\right)^2$



$$A = A_1 - \overset{1}{A_{Segment}} - \overset{2}{A_\Delta} - \overset{3}{A_{Sektor}}$$

$$= A_1 - \left(\frac{\alpha\pi}{360°} - \frac{\sin\alpha}{2}\right) r_1^2 - \frac{1}{2} r_3^2 \cdot \sin\beta - \frac{\beta^*}{360°} \cdot r_3^2 \pi$$

$\beta^* = 360° - \beta$

$$= 5^2 \pi - \left(\frac{73{,}74°}{360°}\pi - \frac{\sin 73{,}74°}{2}\right) 5^2 - \frac{1}{2}\left(\frac{25}{8}\right)^2 \cdot \sin 147{,}5° - \frac{212{,}5°}{360°}\left(\frac{25}{8}\right)^2 \pi$$

$\tan\frac{\alpha}{2} = \frac{3}{4}$

$\alpha = 36{,}87° \cdot 2$

$= 73{,}74°$

$\sin\frac{\beta}{2} = \frac{3}{25/8}$

$\beta = 147{,}5°$

$= 78{,}54 - 4{,}09 - 2{,}62 - 18{,}11$

$= \underline{53{,}7}$

Passilla
VI15

4. $f(x) = ax^2 + \frac{b}{x^2}$

$E(1|1)$ Extremum

a) $f(-x) = a(-x)^2 + \frac{b}{(-x)^2} = ax^2 + \frac{b}{x^2} = f(x)$

b) I $f(1) = 1$

II $f'(1) = 0$ $\quad f'(x) = 2ax - \frac{2b}{x^3}$

I $a + b = 1$

II $2a - 2b = 0$

$a = \frac{1}{2}$

$b = \frac{1}{2}$ $\quad f(x) = \frac{1}{2}x^2 + \frac{1}{2x^2}$ $\quad f'(x) = x - \frac{1}{x^3}$

c) $\frac{f(x) - 0}{x - 0} = x - \frac{1}{x^3}$

$\frac{1}{2}x^2 + \frac{1}{2x^2} = x^2 - \frac{1}{x^2}$

$\frac{1}{2}x^2 - \frac{3}{2x^2} = 0$

$x^4 - 3 = 0$

$x = \pm \sqrt[4]{3}$ $\quad B(\pm \sqrt[4]{3} \mid \frac{7}{6}\sqrt{3})$

$y = \frac{1}{2}\sqrt{3} + \frac{1}{2\sqrt{3}} = \frac{1}{2}\sqrt{3} + \frac{1}{6}\sqrt{3} = \frac{7}{6}\sqrt{3}$

$m = \pm\frac{\frac{7}{6}\sqrt{3}}{\sqrt[4]{3}} = \pm\frac{7}{6}\sqrt[4]{3}$

5, 9 Zettel : 1 .. 9   3 Z. o. Z.

Passella F15

a) $P(\text{nur ungerade Ziffern}) = \frac{5 \cdot 4 \cdot 3}{9 \cdot 8 \cdot 7} = \frac{5}{43} = 11{,}9\%$

b) $P(\text{mind. ein Z. ungerade})$

$= 1 - P(\text{keine Z. ungerade})$

$= 1 - \frac{4 \cdot 3 \cdot 2}{9 \cdot 8 \cdot 7} = \frac{20}{21} = 95{,}2\%$

c) A : letzte Ziffer ist ungerade

B : mindestens eine ungerade Ziffer

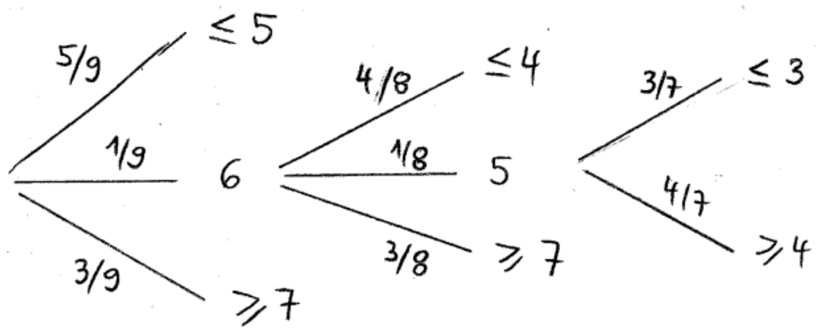
$$P_B(A) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(uuu) + P(ugu) + P(guu) + P(ggu)$$

$$= \frac{5 \cdot 4 \cdot 3}{9 \cdot 8 \cdot 7} + \frac{5 \cdot 4 \cdot 4}{9 \cdot 8 \cdot 7} + \frac{4 \cdot 5 \cdot 4}{9 \cdot 8 \cdot 7} + \frac{4 \cdot 3 \cdot 5}{9 \cdot 8 \cdot 7} = \frac{280}{504} = \frac{5}{9}$$

$$\rightarrow P_B(A) = \frac{5/9}{20/21} = \frac{7}{12} = \underline{\underline{58.3\,\%}}$$

d)



P(Zahl ist grösser als 653) =

$$= \frac{3}{9} + \frac{1}{9} \cdot \frac{3}{8} + \frac{1}{9} \cdot \frac{1}{8} \cdot \frac{4}{7} = \underline{\underline{38{,}3\,\%}}$$