Pass F14

1. Da von verschiedenen Buchstaben die Rede ist, handelt es sich wahrscheinlich um Ziehen ohne Zurücklegen

a) $P(A, B \text{ oder } C) = \frac{1}{9} \cdot \frac{1}{8} \cdot \frac{1}{7} = \frac{1}{504} = 1{,}98\%$ (Ast)

Anzahl Ziehungsmöglichkeiten (A,B,C), (A,C,B) .. (C,B,A) : 6

$P_{gesamt}(A, B \text{ oder } C) = 6 \cdot \frac{1}{504} = \frac{1}{84} = 1{,}19\%$

b) $P(A, \cancel{E}\text{-}I, C\text{-}I) = \frac{1}{9} \cdot \frac{\cancel{7}}{8} \cdot \frac{\cancel{5}}{7}$ (Ast)

B
C analog "

Anzahl Mgl. (A, C-I, C-I) (C-I, A, C-I) (C-I, C-I, A) : 3

$P_{gesamt}(\text{genau } A, B \text{ oder } C) = 3 \cdot 3 \cdot \frac{1}{9} \cdot \frac{\cancel{7}}{8} \cdot \frac{\cancel{5}}{7} = \frac{15}{28} = 53{,}57\%$

c) $P(\bar{E}\bar{E}\bar{E}) = \frac{8}{9} \cdot \frac{7}{8} \cdot \frac{6}{7} = \frac{2}{3}$

$P((\bar{E}\bar{E}\bar{E})\ 10\text{ mal}) = \left(\frac{2}{3}\right)^{10} = 1{,}73\%$

d) $P(E \text{ in } n\text{-Wdh mind einmal}) > 0{,}99$

$1 - P(E \text{ in } n \text{ Wdh nie}) > 0{,}99$

$\left(\frac{2}{3}\right)^n < 0{,}01$

$n > \log_{\frac{2}{3}} 0{,}01 = 11{,}36$

Ab 12 Wiederholungen

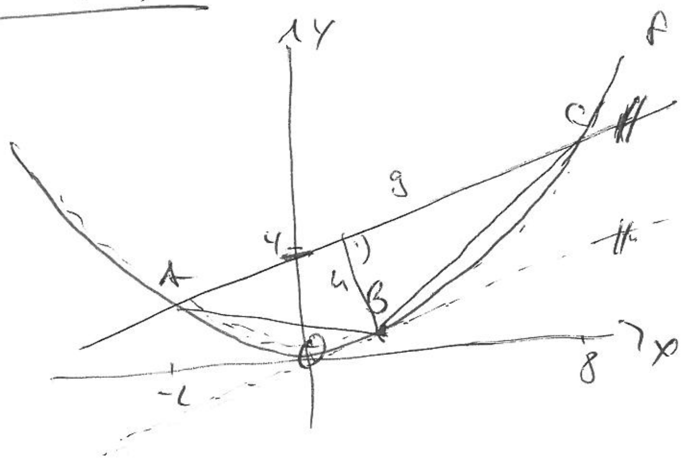
2. $f(x) = \frac{1}{4}x^2$, $g(x) = \frac{3}{2}x + 4$

Pass F14

a) $f = g$

$x_1 = -2$ $\quad y_1 = 1$ $\quad$ A $(-2|1)$

$x_2 = 8$ $\quad y_2 = 16$ $\quad$ C $(8|16)$



b) Steigung von f bei B gleich $\frac{3}{2}$

$f'(x) = \frac{1}{2}x$ $\qquad f'(x) = \frac{3}{2}$

$x = 3$

$y = \frac{9}{4}$

$B(3|\frac{9}{4})$

$AC = \sqrt{10^2 + 15^2} = 5\cdot\sqrt{13}$

g: $y = \frac{3}{2}x + 4$

$\frac{3x - 2y + 8}{\sqrt{13}} = 0$

$B(3|\frac{9}{4})$: $h = \left|\frac{3\cdot 3 - 2\cdot\frac{9}{4} + 8}{\sqrt{13}}\right| = \frac{25}{2\sqrt{13}}$

$A_{ABC} = \frac{1}{2}gh = \frac{1}{2}\cdot 5\sqrt{13}\cdot\frac{25}{2\sqrt{13}} = \frac{125}{4} = J_\Delta$

c) $J_s = \int_{-2}^{8}(g-f)\,dx = \left[\frac{3}{4}x^2 + 4x - \frac{1}{12}x^3\right]_{-2}^{8} = \frac{125}{3}$

$\frac{J_\Delta}{J_s} = \frac{3}{4}$

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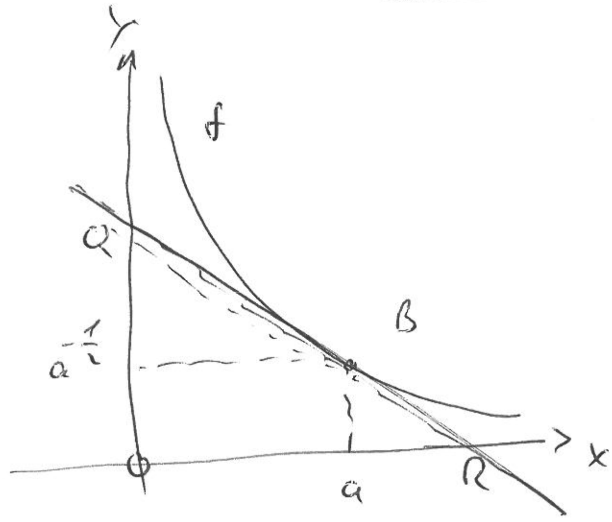
3. $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$B(a \mid f(a)) \cap x\text{-Achse} \to R$

$\cap\ y\text{-Achse} \to Q$



a) $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$

$t:\ y = -\frac{1}{2}a^{-\frac{3}{2}}(x-a) + a^{-\frac{1}{2}}$

$t:\ y = -\frac{1}{2}a^{-\frac{3}{2}}x + \frac{3}{2}a^{-\frac{1}{2}}$

$Q(0 \mid \frac{3}{2}a^{-\frac{1}{2}})$

$R:\quad 0 = -\frac{1}{2}a^{-\frac{3}{2}}x + \frac{3}{2}a^{-\frac{1}{2}}$

$x = 3a$

$R(3a \mid 0)$

b) $QR^2 = (3a)^2 + (\frac{3}{2}a^{-\frac{1}{2}})^2 = 9a^2 + \frac{9}{4}a^{-1}$

$QR = \sqrt{QR^2}$ wird extr. wo QR^2 extr. wird (Monotonie Wurzel)

$z = 9a^2 + \frac{9}{4}a^{-1} \qquad a > 0$

$z' = 18a - \frac{9}{4}a^{-2}$

$z'' = 18 + \frac{9}{2}a^{-3}$

$z' = 0$

$a^3 = \frac{1}{8}$

$a = \frac{1}{2}$

$z''(\frac{1}{2}) > 0 \Rightarrow$ Min

$z(\frac{1}{2}) = \frac{27}{4} = d^2$

$d = \frac{3}{2}\sqrt{3}$

Ränder (!):

$\lim_{a \to 0} z(a) = \infty$

$\lim_{a \to \infty} z(a) = \infty$

Kein Min

PassF14

4. g_1: $A(-4|1)$ $B(0|3)$

$g_2 \perp u: 9x+10y+50=0$, $C(1|-2)$

a) g_1: $\underline{y} = \frac{2}{4}x + 3 = \underline{\frac{1}{2}x + 3}$

g_2: Normalenvektor von u ist Richtungsvektor g_2

$\vec{n} = \binom{9}{10}$

g_2: $\vec{x} = \binom{1}{-2} + t\binom{9}{10}$

$g_1 \cap g_2$:

$-2 + 10t = \frac{1}{2}(1+9t) + 3$

$t = 1$

$\underline{\underline{S(10|8)}}$

$\vec{v_1} = \binom{2}{1}$

$\vec{v_2} = \binom{9}{10}$

$\cos\alpha = \frac{\binom{2}{1}\binom{9}{10}}{\sqrt{5}\sqrt{181}} = \frac{18+10}{\sqrt{5}\sqrt{181}} = 0{,}93075$

$\underline{\underline{\alpha = 21{,}45°}}$

b) u: $y = \frac{8}{10}x = \frac{4}{5}x$

$\vec{w} = \frac{1}{v_1}\vec{v_1} + \frac{1}{v_2}\vec{v_2} = \frac{1}{\sqrt{5}}\binom{2}{1} + \frac{1}{\sqrt{181}}\binom{9}{10} =$

$\vec{u} = \binom{5}{4}$

$\left.\begin{array}{l} \cos\alpha_1 = \frac{\binom{2}{1}\binom{5}{4}}{\sqrt{5}\sqrt{41}} = \frac{10+4}{\sqrt{5}\sqrt{41}} \approx 0{,}9778 \\ \cos\alpha_2 = \frac{\binom{9}{10}\binom{5}{4}}{\sqrt{181}\sqrt{41}} = \frac{45+40}{\sqrt{181}\sqrt{41}} \approx 0{,}987 \end{array}\right\} \neq$

Also sind die Winkel auch verschieden, wenn auch nur knapp.

5. $f(x) = x + \sqrt{x^2-16}$ PassF14

$\mathbb{D}$: $x^2 - 16 \geq 0$

$x^2 \geq 16$

$|x| \geq 4$

$\mathbb{D} =]-\infty; -4] \cup [4; \infty[$

$= \mathbb{R} \setminus]-4; 4[$

NST:

$x = -\sqrt{x^2-16}$

$x^2 = x^2 - 16$

$0 = -16$ (f)

, also kein Lsg.

$f'(x) = 1 + \frac{x}{\sqrt{x^2-16}}$ $\quad \mathbb{D}_{f'} = \mathbb{R} \setminus [-4; 4]$

also vert. Tangente bei $x = \pm 4$

$f'(x) = 0$

$\sqrt{x^2+16} = +x$

$x^2 + 16 = x^2$

$16 = 0$ (f)

Kein waag. T.

Randextrema bei $x = \pm 4$

$\left.\begin{array}{l} \text{Min } (+4|4) \\ \text{Max } (-4|-4) \end{array}\right\} W = [-4; 0[\cup [4; \infty[$

Umkehrfunktion ist nicht im Reglement.

$y = x + \sqrt{x^2-16}$

$x \leftrightarrow y$: $x = y + \sqrt{y^2-16}$

$(x-y)^2 = y^2 - 16$

$x^2 - 2xy + y^2 = y^2 - 16$

$\bar{f}$: $y = \frac{x^2+16}{2x} = \frac{1}{2}x + \frac{8}{x}$