Pass F12

1. $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

$A(0|5)$: I. $f(0) = \underline{5 = e}$

$V(-1|0)$: II $f(-1) = 0 = a - b + c - d + 5$

III $f''(-1) = 0 = 12a - 6b + 2c$

$W(1|0)$: IV $f(1) = 0 = a + b + c + d + 5$

V $f''(1) = 0 = 12a + 6b + 2c$

$\underline{a = 1}$

$\underline{b = 0}$

$\underline{c = -6}$

$\underline{d = 0}$

$\underline{f(x) = x^4 - 6x^2 + 5}$

$f'(x) = 4x^3 - 12x = 0$

$\underbrace{4x}\,\underbrace{(x^2 - 3)} = 0$

$\underline{x = 0}$ $\underline{x = \pm\sqrt{3}}$

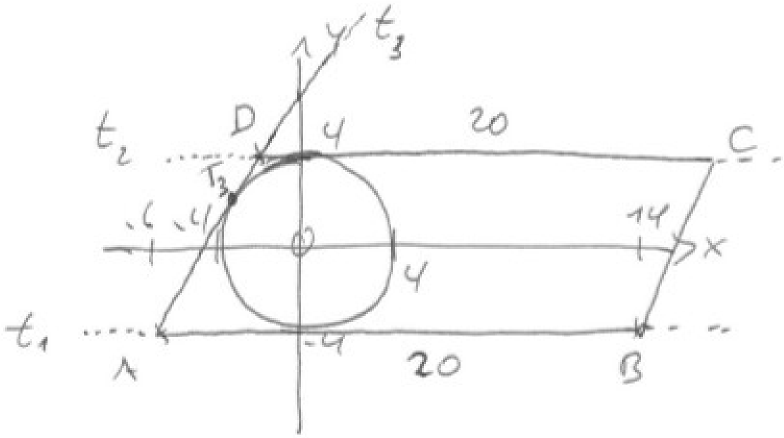
$\underline{y = 5}$ y = -4

Min, da $f''(\pm\sqrt{3}) > 0$

Aufgrund der Achsensymmetrie (nur gerade Exponenten) sind die lokalen Minima auch absolute Minima.

Somit ist die Wertemenge $\underline{W = [-4; \infty[}$

2.



$t_1: y = -4$

$t_2: y = +4$

A(-6|-4) Kreispolarsierung: $-6x - 4y = 16$

$y = -\frac{3}{2}x - 4$

in Kreis: $x^2 + (-\frac{3}{2}x - 4)^2 = 0$

$x_1 = 0$

$x_2 = -48/13$ $\quad y = 20/13$

$T_3(-\frac{48}{13} | \frac{20}{13})$

$t_3: y = \frac{20/13 - (-4)}{-48/13 - (-6)} \cdot (x + \frac{48}{13}) + \frac{20}{13}$

$y = \frac{12}{5}x + \frac{52}{5}$

$t_3 = 4$ $\quad D(-\frac{8}{3} | 4)$

$x = -\frac{8}{3}$

$AD = \sqrt{8^2 + (-\frac{8}{3} - (-6))^2} = 26/3$

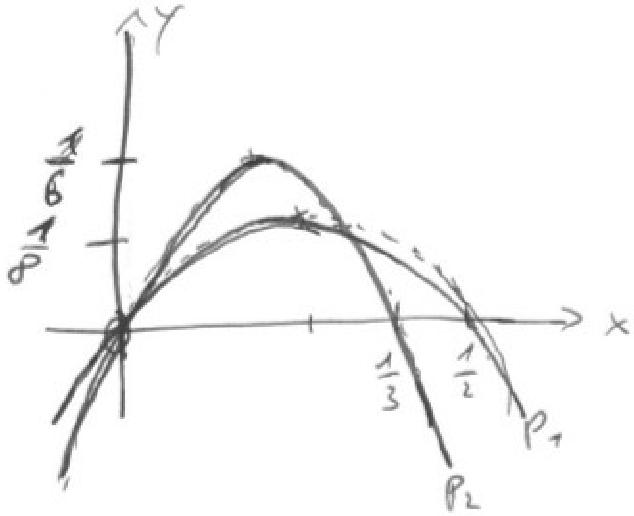
$U = 2 \cdot \frac{26}{3} + 40 = \frac{172}{3}$

Pass F12

3. $f_a(x) = a\left(x - (a+1)x^2\right) = ax\left(1-(a+1)x\right)$

a) $f_1(x) = x - 2x^2 = x(1-2x)$

$f_2(x) = 2x - 6x^2 = 2x(1-3x)$



b) $f_a(x) = 0$

$\underbrace{ax}\,(\underbrace{1-(a+1)x}) = 0$

$\underline{x=0}$ $\quad$ $\underline{x = \frac{1}{a+1}}$

Mitte

$x_S = \frac{1}{2(a+1)}$ $\quad$ $y_S = \frac{a}{4(a+1)}$ $\qquad$ $\underline{S\left(\frac{1}{2(a+1)} \,\middle|\, \frac{a}{4(a+1)}\right)}$

$x = \frac{1}{2(a+1)}$

$a = \frac{1}{2x} - 1 \longrightarrow \underline{\underline{y}} = \frac{\frac{1}{2x}-1}{4\left(\frac{1}{2x}-1+1\right)} = \frac{\frac{1-2x}{2x}}{4\,\frac{1}{2x}} = \frac{1}{4}(1-2x) = \underline{\underline{-\frac{1}{2}x + \frac{1}{4}}}$

Ortskurve der Scheitel

c) $A = \int_0^{\frac{1}{a+1}} f_a(x)\,dx = \left[\frac{a}{2}x^2 - a\frac{a+1}{3}x^3\right]_0^{\frac{1}{a+1}}$

$= \frac{a}{2}\cdot\frac{1}{(a+1)^2} - a\frac{a+1}{3}\,\frac{1}{(a+1)^3} - 0$

$\underline{\underline{A = \frac{a}{6(a+1)^2}}}$

Pass F12

4. $\lim\limits_{x \to \infty} \frac{2e^{2x}}{1+e^{2x}} = \lim\limits_{x \to \infty} \frac{2}{\underbrace{e^{-2x}}_{\to 0}+1} = \underline{\underline{2}}$

$$f'(x) = 2 \; \frac{e^{2x} \cdot 2(1+e^{2x}) - e^{2x} e^{2x} \cdot 2}{(1+e^{2x})^2}$$

$$= 4 \; \frac{e^{2x} + e^{4x} - e^{4x}}{(1+e^{2x})^2}$$

$$f'(x) = 4 \; \frac{e^{2x}}{(1+e^{2x})^2}$$

$$f''(x) = 4 \cdot \frac{e^{2x} \cdot 2\,(1+e^{2x})^2 - e^{2x} \cdot 2(1+e^{2x}) \cdot e^{2x} \cdot 2}{(1+e^{2x})^4}$$

$$= 8 \; \frac{e^{2x}(1+e^{2x}) - e^{4x} \cdot 2}{(1+e^{2x})^3}$$

$$= 8 \; \frac{e^{2x} - e^{4x}}{(1+e^{2x})^3}$$

$f''(x) = 0$

$e^{2x} = e^{4x}$

$2x = 4x$

$\underline{x = 0}$ $\quad \underline{y = 1}$ $\quad WP(0|1)$ $\Big\}$ $\underline{\underline{y = x+1}}$

$f'(0) = 1$

Wendetangente

5. PassF12

a) A in zwei Würfen am Ziel: 6+6; 5+6; 6+5; 6+4; 4+6; 5+5: $\underline{:6}$

$\underline{P(A)} = \frac{1}{36} \cdot 6 = \underline{\frac{1}{6}} = \underline{16,7\%}$

B " " : 6+6; 5+6; 6+5 .. wie A : 6

und: 6+3; 3+6; 5+4; 4+5; :4

$\underline{10}$

$\underline{P(B)} = \frac{1}{36} \cdot 10 = \underline{\frac{5}{18}} = \underline{27,8\%}$

b)

$\frac{1}{6}$ A im Ziel — $\frac{13}{18}$ B nicht Z ⇐ $\overbrace{\frac{1}{6} \cdot \frac{13}{18}}^{13/108}$

A im Ziel — B Ziel

$\frac{5}{6}$ A nicht Ziel — $\frac{5}{18}$ B Ziel ⇐ $\frac{5}{6} \cdot \frac{5}{18}$

A nicht Ziel — B nicht Z

$\left.\right\} \underline{\underline{\frac{19}{54}}} = \underline{\underline{35,2\%}}$

c) P(genau einer im Ziel) $= \frac{19}{54}$ (siehe b)

P(genau A im Ziel) $= \frac{13}{108}$

$\left.\right\} \frac{13/108}{19/54} = \frac{13}{38} = \underline{34,2\%}$