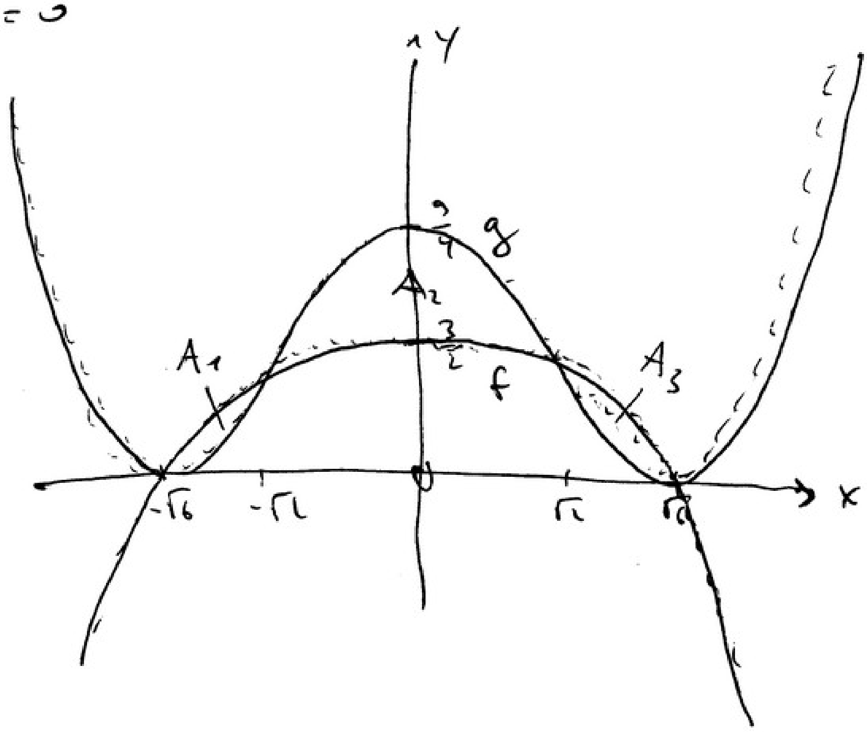
Pass F11

1. $f(x) = f^2(x)$

$\underbrace{f(x)}_{=0}(\underbrace{f(x)-1}_{=0}) = 0$

$x = \pm\sqrt{6}$ $\quad x = \pm\sqrt{2}$

$y = 0$ $\quad y = 1$




Beide Funktionen sind achsensymmetrisch (nur gerade Exp.) somit $A_1 = A_3$ und $A_2$ symm. zu y-Achse:

$$A = 2\int_{\sqrt{2}}^{\sqrt{6}} (f-g)\,dx + 2\int_0^{\sqrt{2}} (g-f)\,dx$$

$$= 2\cdot\int_{\sqrt{2}}^{\sqrt{6}} \left(-\frac{1}{16}x^4 + \frac{1}{2}x^2 - \frac{3}{4}\right)dx - 2\int_0^{\sqrt{2}}\left(-\frac{1}{16}x^4 + \frac{1}{2}x^2 - \frac{3}{4}\right)dx$$

$$= 2\left[-\frac{1}{80}x^5 + \frac{1}{6}x^3 - \frac{3}{4}x\right]_{\sqrt{2}}^{\sqrt{3}} - 2\left[-\frac{1}{80}x^5 + \frac{1}{6}x^3 - \frac{3}{4}x\right]_0^{\sqrt{2}}$$

$$= 2\cdot\left(\frac{7}{15}\sqrt{2} - \frac{1}{5}\sqrt{6}\right) - 2\left(-\frac{7}{15}\sqrt{2}\right)$$

$$= \underline{\underline{\frac{28}{15}\sqrt{2} - \frac{2}{5}\sqrt{6}}} \approx 1{,}66 \quad \text{größte Gesamtfläche}$$

oder, wenn nur $A_2$ gemeint ist: $\underline{\underline{\frac{14}{15}\sqrt{2}}} \approx 1{,}32$

PASS F11

2a)

$$\lim_{h\to 0} \frac{\sqrt{(x+h)^2+5}-\sqrt{x^2+5}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{(x+h)^2+5}-\sqrt{x^2+5}\right)\left(\sqrt{(x+h)^2+5}+\sqrt{x^2+5}\right)}{h\cdot\left(\sqrt{(x+h)^2+5}+\sqrt{x^2+5}\right)}$$

$$= \lim_{h\to 0} \frac{(x+h)^2+5-(x^2+5)}{h\cdot(\text{"} \quad \text{"} \quad )}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h\cdot(\ \text{-}\ )} = \lim_{h\to 0} \frac{2x+\overset{\to 0}{h}}{\underbrace{\sqrt{(x+h)^2+5}}_{\to\sqrt{x^2+5}}+\sqrt{x^2+5}} = \frac{2x}{2\sqrt{x^2+5}} = \underline{\underline{\frac{x}{\sqrt{x^2+5}}}}$$

b) $f(x) = bx^3 - x^2$

$f'(x) = 3bx^2 - 2x = 0$

$\underbrace{x}\,\underbrace{(3bx-2)} = 0$

$x_1 = 0 \quad x_2 = \frac{2}{3b}$

$y_1 = 0 \quad y_2 = -\frac{4}{27b^2} < 0$

$y_1 - y_2 = 1$

$\frac{4}{27b^2} = 1$

$\underline{\underline{b = \pm\frac{2}{9}\sqrt{3}}}$

Pass F 11

3. Gesamtzahl 3 verschiedene Punkte auswählen: $9 \cdot 8 \cdot 7 = 504$

a) erste Punkt: 9 Möglichkeiten
zweite " : 4 "
dritte " : 1 "

$\Big\}\ 36$ ; $P = \frac{36}{504} = \frac{1}{14} = 7{,}14\,\%$

b) Anzahl Möglichkeiten nicht 1, C, 6 oder 3 zu wählen: $5 \cdot 4 \cdot 3 = 60$ ; $P = \frac{60}{504} = \frac{5}{42} = 11{,}9$

c) $\equiv \quad ||| \quad \diagdown \quad \diagup$ 8 Geraden

jede Gerade kann auf $\underbrace{3 \cdot 2 \cdot 1}_{6}$ Möglichkeiten gebildet werden

$\Big\}\ 48$ ; $P = \frac{48}{504} = \frac{2}{21} = 9{,}52$

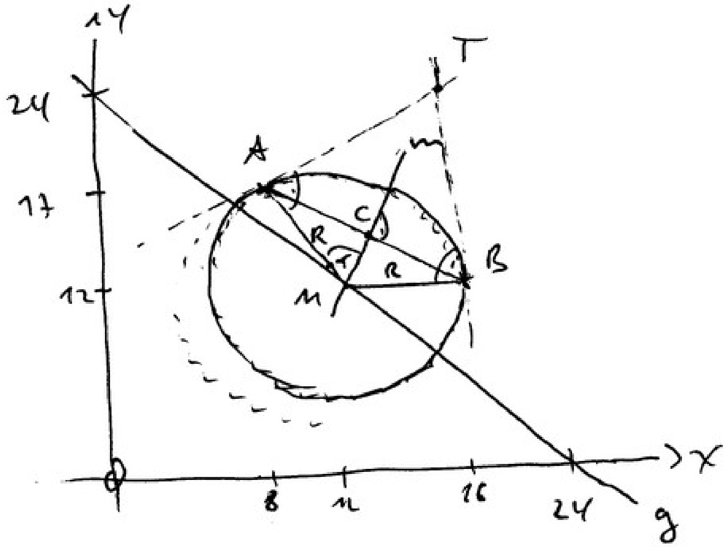
d) $P(E) = \frac{1}{9}$ $\quad$ P(in n Versuchen E mind. einmal)

$P(\bar{E}) = \frac{8}{9}$ $\quad = 1 - P$(in n Versuchen E keinmal)

$$= 1 - \left(\frac{8}{9}\right)^n = 1 - \left(\frac{8}{9}\right)^{15} = 82{,}9\,\%$$

4.






Mittelsenkrechte zu AB: $\vec{r}_C = \frac{\vec{r}_A + \vec{r}_B}{2} = \begin{pmatrix}12\\15\end{pmatrix}$ ; $\vec{AB} = \begin{pmatrix}8\\-4\end{pmatrix} \sim \begin{pmatrix}2\\-1\end{pmatrix}$

$\hookrightarrow \vec{n} = \begin{pmatrix}1\\2\end{pmatrix}$

$$m: \vec{X} = \begin{pmatrix}12\\15\end{pmatrix} + \lambda \begin{pmatrix}1\\2\end{pmatrix}$$

$m \cap g \to M$: $\quad 12 + \lambda + 15 + 2\lambda - 24 = 0$

$\underline{\lambda = -1}$

$\underline{\underline{M(11|13)}} \in g$ ✓

$\underline{\underline{R}} = MA = \sqrt{3^2+4^2} = \underline{\underline{5}} = MB$

$k: (x-11)^2 + (y-13)^2 = 25$

b) $AC = \frac{1}{2} AB = \frac{1}{2}\sqrt{8^2+4^2} = 2\sqrt{5}$

$\sin\alpha = \frac{2\sqrt{5}}{5} \Rightarrow \quad \alpha = 63{,}435°$

$\varphi = 2\alpha = 126{,}87°$

$\underline{\underline{b_{AB}}} = \pi \frac{\varphi}{180°} \cdot R = \underline{\underline{11{,}1}}$

c) $\underline{\underline{A_{Drach}}} = \frac{1}{2} e f = \frac{1}{2} MT \cdot AB = \frac{1}{2} \cdot 4\sqrt{5} \cdot MT = 2\sqrt{5} \cdot 5\sqrt{5} = \underline{\underline{50}}$

$MT = \frac{R}{\cos\alpha}$

$= \frac{5}{\sqrt{1-\sin^2\alpha}} = \frac{5}{\sqrt{1-\frac{4}{5}}} = 5\sqrt{5}$

$\sin\alpha = \frac{2}{\sqrt{5}}$

Pass F11

5a) $f(x) = 2x\,e^{-2x^2}$

$f'(x) = 2\cdot e^{-2x^2} + 2x\,e^{-2x^2}\cdot(-4x)$

$= 2(1-4x^2)\,e^{-2x^2} = 0$

$x = \pm\frac{1}{2} \quad y = \pm e^{-\frac{1}{2}}$

VZT

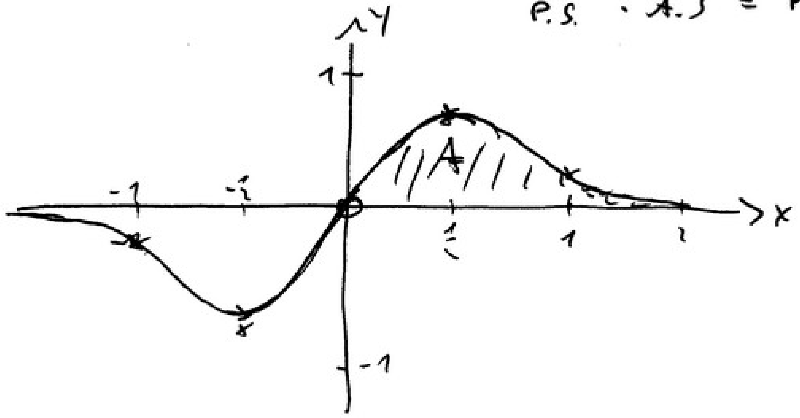
| $x$ | 0 | $\frac{1}{2}$ | 1 |
|---|---|---|---|
| $f'$ | + | 0 | − |

$\nearrow \underset{Max}{\rightarrow} \searrow$

$Max(\frac{1}{2} \mid e^{-\frac{1}{2}})$

$\hookrightarrow$ Symm.: $Min(-\frac{1}{2} \mid -e^{-\frac{1}{2}})$

$\underbrace{2x}_{P.S.} \cdot \underbrace{e^{-2x^2}}_{A.S} = P.S$




$F(x) = k\cdot e^{-2x^2}$

$\left.\begin{array}{l} F'(x) = -4x\,k\,e^{-2x^2} \\ f(x) = 2x\;e^{-2x^2} \end{array}\right\}\; k = -\frac{1}{2} \qquad \underline{F(x) = -\frac{1}{2}e^{-2x^2}}$

$\underline{\underline{A}} = \lim_{a\to\infty}\int_0^a f(x)\,dx = \lim_{a\to\infty}\left[F(x)\right]_0^a = \lim_{a\to\infty}\left(\underbrace{-\frac{1}{2}e^{-2a^2}}_{\to 0} + \frac{1}{2}\right) \underline{\underline{= \frac{1}{2}}}$

b) $f(x) = ax\,e^{bx^2}$

$f'(x) = a e^{bx^2} + ax e^{bx^2}\cdot 2bx$

$= a(1+2bx^2)\,e^{bx^2}$

$H(2|2)$: I. $f(2) = 2$

II. $f'(2) = 0$

I. $2a\,e^{4b} = 2$

II. $a\underbrace{(1+8b)}_{b = -\frac{1}{8}} e^{4b} = 0$

$\underline{\underline{b = -\frac{1}{8}}}$

in I: $\underline{\underline{a = e^{1/2}}}$

$\underline{\underline{f(x) = \sqrt{e}\cdot x\cdot e^{-\frac{1}{8}x^2}}}$

Pass F11

6. a) $|\vec{a}| = 2|\vec{b}| \Rightarrow a = 2b$

$(\vec{a}+\vec{b})(\vec{a}+2\vec{b}) = 0$

$a^2 + 2\vec{a}\vec{b} + \vec{a}\vec{b} + 2b^2 = 0$

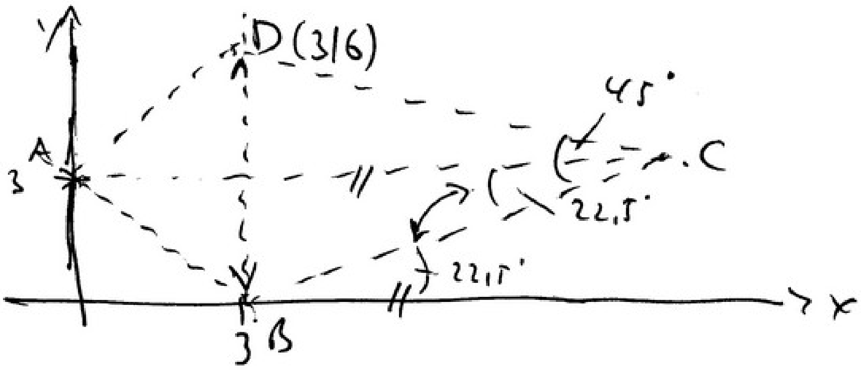
$\vec{a}\vec{b} = \dfrac{-a^2-2b^2}{3}$

$a \cdot b \cdot \cos\alpha = \dfrac{-a^2-2b^2}{3}$

$\cos\alpha = \dfrac{-a^2-2b^2}{3ab} = \dfrac{-4b^2-2b^2}{3\cdot 2b\cdot b} = -\dfrac{6}{6} = -1$

$\underline{\underline{\alpha = 180°}}$ $\quad$ $\underline{\underline{\vec{a} = -2\vec{b}}}$

b)




Wegen Symmetrie: $B \rightarrow D(3|6)$

y-Koordinate von C: 3 (von A)

Steigung von BC: $\tan 22{,}5° = 0{,}4142 = m$

BC: $y = m\cdot(x-3)+0 = 3$

$\underline{x} = \dfrac{3}{m} + 3 = \underline{10{,}243}$

$C(10{,}243|3)$

$A = \frac{1}{2}\, BD \cdot AC = \frac{1}{2}\cdot 6 \cdot 10{,}243 = \underline{\underline{30{,}73}}$