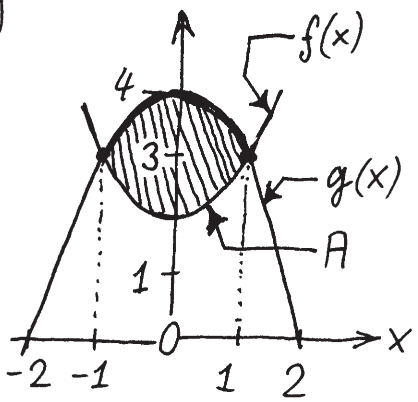
SMK Passerelle-Prüfung Winter 2009 Mathematik www.mathepauker.com

1.a)



$f \cap g: x^2+2 = 4-x^2 \rightarrow x = \pm 1 \rightarrow S_1\binom{-1}{3}$ und $S_2\binom{1}{3}$

$f'(x) = 2x \rightarrow f'(1) = 2$

$g'(x) = -2x \rightarrow g'(1) = -2$

$\alpha = |\arctan(2) - \arctan(-2)| = 2\arctan 2 = 127°$

$A = \int_{-1}^{1}[g(x)-f(x)]dx = \int_{-1}^{1}(-2x^2+2)dx$

$= \left(-\frac{2}{3}x^3+2x\right)_{-1}^{1} = -\frac{2}{3}+2-\left(\frac{2}{3}-2\right) = \frac{8}{3}$

b) $f \cap g: x^2+k = 2k-x^2 \rightarrow x^2 = k/2$

$f'(x)\cdot g'(x) = 2x\cdot(-2x) = -4x^2 = -4\cdot k/2 = -2k = -1 \rightarrow k = 1/2$

2.a) HNF von t: $\frac{3x-4y+46}{\sqrt{3^2+4^2}} = \frac{3x-4y+46}{5} = 0 \rightarrow$

$\left|\frac{3\cdot 12-4\cdot 8+46}{5}\right| = \left|\frac{50}{5}\right| = 10 = r \rightarrow k: (x-12)^2+(y-8)^2-10^2 = 0$

$k: x^2+y^2-24x-16y+108 = 0$

b) $A_2 = A_1/4 \rightarrow r_2 = r_1/2 = 5 \rightarrow r_2 = 5$

HNF von t_2: $\frac{3x-4y+q}{5} = 0 \rightarrow \frac{3\cdot 12-4\cdot 8+q}{5} = \pm r_2 = \pm 5 \rightarrow$

$$4+q = \pm 25 \begin{cases} \oplus \rightarrow q_1 = 25-4 = 21 \longrightarrow t_{21}: 3x-4y+21 = 0 \\ \ominus \rightarrow q_2 = -25-4 = -29 \longrightarrow t_{22}: 3x-4y-29 = 0 \end{cases}$$

3.) $f'(x) = -\frac{2ax}{b}e^{-x^2/b}$

$f''(x) = -\frac{2a}{b}\left[1-\frac{2x^2}{b}\right]e^{-x^2/b}$

$f''(-\sqrt{2}) = -\frac{2a}{b}\left[1-4/b\right]e^{-2/b} = 0 \rightarrow b = 4$

$f(-\sqrt{2}) = a\cdot e^{-2/b} = a\cdot e^{-1/2} = a/\sqrt{e} = 5/\sqrt{e} \rightarrow a = 5$

4a)
I. P(ungleichfarbig)=1-P(gleichfarbig)

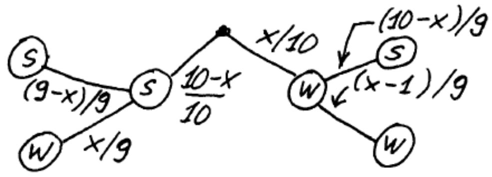
Behauptung:
2P(gleichfarbig)=P(ungleichrfarbig)
Mit I. :
P(gleichfarbig)=1/3

P(gleichfarbig)=P(r,r)+P(b,b)+P(w,w)+P(s,s)=

$$\frac{2\cdot 1}{20\cdot 19}+\frac{2\cdot 1}{20\cdot 19}+\frac{6\cdot 5}{20\cdot 19}+\frac{10\cdot 9}{20\cdot 19}=\frac{124}{380}=\frac{31}{95}\neq\frac{1}{3}$$

Die Behauptung ist also (knapp) falsch.

b)



$$P_{gleich} = P_{ungleich} - 1/15$$

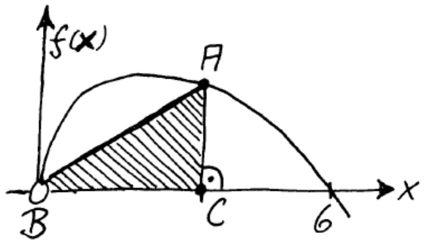
$$\frac{x\cdot(x-1)}{90}+\frac{(10-x)\cdot(9-x)}{90}=\frac{x\cdot(10-x)}{90}+\frac{(10-x)\cdot x}{90}-\frac{6}{90}$$

$$x^2-x+x^2-19x+90=10x-x^2+10x-x^2-6$$

$$\rightarrow x^2-10x+24=(x-6)\cdot(x-4)=0$$

1. Lösung: $x_1=6$

2. " : $x_2=4$



5.) $A\binom{x}{f(x)}$, $B\binom{0}{0}$ und $C\binom{x}{0}$

$$A_\Delta=\frac{x\cdot f(x)}{2}=\frac{x(6-x)\sqrt{x}}{2}=3x^{3/2}-\frac{x^{5/2}}{2}$$

$$\frac{dA_\Delta}{dx}=\frac{9}{2}x^{1/2}-\frac{5}{4}x^{3/2}=\frac{\sqrt{x}}{2}\left(9-\frac{5}{2}x\right)=0\rightarrow x=\frac{18}{5}=3.6$$

$$A\binom{3.6}{2.4\cdot\sqrt{3.6}}\rightarrow A\binom{3.6}{4.554},\ B\binom{0}{0} \text{ und } C\binom{3.6}{0}$$

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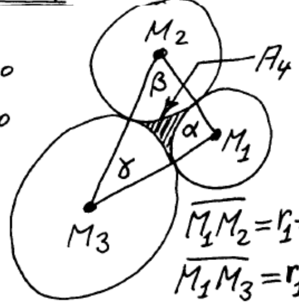
6.) $\alpha=\arccos[(6^2+8^2-10^2)/(2\cdot 6\cdot 8)]=\arccos(0)=90°$

$\rightarrow\beta=\arcsin\left(\frac{8}{10}\right)=53.13°$ und $\gamma=90°-\beta=36.87°$



$\overline{M_1M_2}=r_1+r_2=6$

$\overline{M_1M_3}=r_1+r_3=8$

$\overline{M_2M_3}=r_2+r_3=10$

Kreissektor um M_1: $A_1=\frac{\pi r_1^2}{4}=\frac{\pi\cdot 2^2}{4}=\pi=\underline{3.1416}$

" " M_2: $A_2=\frac{\pi r_2^2\cdot\beta}{360°}=\frac{\pi\cdot 4^2\cdot 53.13°}{360°}$

" " M_3: $A_3=\frac{\pi\cdot r_3^2\cdot\gamma}{360°}=\frac{\pi\cdot 6^2\cdot 36.87°}{360°}$

$A_4=\frac{1}{2}\cdot\overline{M_1M_2}\cdot\overline{M_1M_3}-A_1-A_2-A_3$

$A_2=7.4184$

$A_3=11.5830$

$A_4=1.8570$