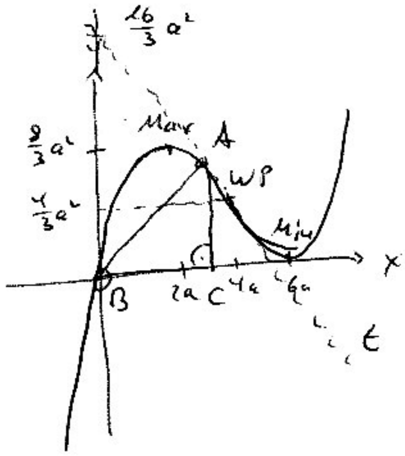
Matura H10, GF



1. $a > 0$; $f_a(x) = \frac{1}{12a}x^3 - x^2 + 3ax$

a) $f_a'(x) = \frac{1}{4a}x^2 - 2x + 3a$

$f_a''(x) = \frac{1}{2a}x - 2$

NST.: $f_a(x) = 0$

$\underline{x_1 = 0}$ einfach

$\underline{x_{2/3} = 6a}$ doppelt, kein Vzw, Extr.

Extr.: $f_a'(x) = 0$

$x_1 = 6a$ $y_1 = 0$ $f''(6a) = 1 > 0 \Rightarrow \underline{Min\ (6a|0)}$

$x_2 = 2a$ $y_2 = \frac{8}{3}a^2$ $f''(2a) = -1 < 0 \rightarrow \underline{Max\ (2a|\frac{8}{3}a^2)}$

WP: $f''(x) = 0$

$x = 4a$ $y = \frac{4}{3}a^2$

└ einfach, Vzw, $\underline{WP(4a|\frac{4}{3}a^2)}$

b) $A = 18 = \int_0^{6a} f(x)\,dx = \left[\frac{1}{48a}x^4 - \frac{1}{3}x^3 + \frac{3a}{2}x^2\right]_0^{6a}$

$18 = 9a^3$

$\underline{a = \sqrt[3]{2}}$

c) t: $m = f'(4a) = -a$ t: $y = -a(x - 4a) + \frac{4}{3}a^2$

$y = -ax + \frac{16}{3}a^2$

$A = \int_0^{4a} (t(x) - f(x))\,dx = \int_0^{4a} \left(-\frac{1}{12a}x^3 + x^2 - 4ax + \frac{16}{3}a^2\right)dx$

$\underline{A} = \left[-\frac{1}{48a}x^4 + \frac{1}{3}x^3 - 2ax^2 + \frac{16}{3}a^2x\right]_0^{4a} = \underline{\frac{16}{3}a^3}$

d) $\underline{a = 2}$

$A = \frac{1}{2}x \cdot f_2(x) = \frac{1}{48}(x^4 - 24x^3 + 144x^2)$ $x \in [0; 12]$

$A'(x) = \frac{1}{12}(x^3 - 18x^2 + 72x) = 0$

$x_1 = 0$; $x_2 = 6$; $x_3 = 12$

$A(0) = A(12) = 0$

$A(6) = 27$

$A''(x) = \frac{1}{4}x^2 - 3x + 6$ $A''(6) = -3 \Rightarrow$ Max ~~[illegible]~~ $\underline{A(6|9)}$

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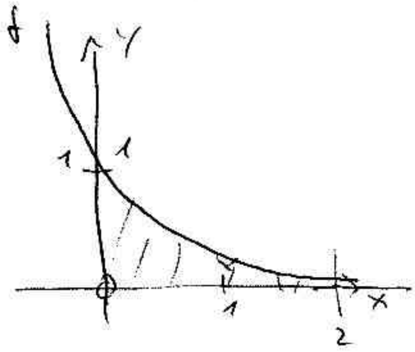

2. $f(x) = e^{-2x}$

a) $A = \int_0^2 f(x)\,dx = \left[-\frac{1}{2}e^{-2x}\right]_0^2$

$\underline{A} = -\frac{1}{2}e^{-4} + \frac{1}{2} = \underline{\frac{1}{2} - \frac{1}{2}e^{-4}} \approx 0{,}491$

b) berühren: I. $f(x) = g(x)$ | $t(x) = -x$

II. $f'(x) = t'(x)$

I $e^{-2x} + k = -x$

II $-2e^{-2x} = -1$

$x = \frac{1}{2}\ln 2$; $\underline{k = -\frac{1}{2}(1 + \ln 2)}$

c) $g(x) = y = e^{-2x} + k$

$x \leftrightarrow y$: $x = e^{-2y} + k$

g^{-1}: $\underline{y = -\frac{1}{2}\ln(x - k)}$

3. Urne 56 Kugeln: x weiß, 2x rot, 56 − 3x schwarz

a) zulässig für x: $56 - 3x \geq 0$

$\underline{x \leq 18}$

b) $\underline{x = 10}$ weiß, 20 rot, 26 schwarz; drei Kugeln gleichzeitig

b1) WS(alle gleichfarbig) = WS(3w) + WS(3r) + WS(3s)

$= \frac{10}{56}\,\frac{9}{55}\,\frac{8}{54} + \frac{20\cdot 19\cdot 18}{56\cdot 55\cdot 54} + \frac{26\cdot 25\cdot 24}{56\cdot 55\cdot 54} = \underline{\frac{193}{1386} = 13{,}9\,\%}$

b2) WS(alle verschieden) $= 6 \cdot \frac{10\cdot 20\cdot 26}{56\cdot 55\cdot 54} = \underline{\frac{130}{693} = 18{,}8\,\%}$

$\left.\begin{matrix} rws \\ rsw \\ swr \\ \vdots \end{matrix}\right\} 6$

3! möglichen Anordnungen

c) gleichzeitig 2 Kugeln, x beliebig

c1) $\underline{WS(\text{gleichfarbig})(x)} = \frac{x\cdot(x-1) + 2x(2x-1) + (56-3x)(55-3x)}{56\cdot 55} = \underline{\frac{1}{3080}(14x^2 - 336x + 3080)}$

c2) WS min/max: WS ist Parabel, nach oben geöffnet → Min absolut Scheitel $x_S = -\frac{b}{2a}$

Min: $x_S = 12$ → da weiter von Symmetrieachse entfernt

Max: Rand von D: $\underline{WS(0) = 1}$

$WS(18) = 0{,}51$

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4. $g: \vec{x} = \underbrace{\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}}_{A} + t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$; $Z(0|0|3)$ in E

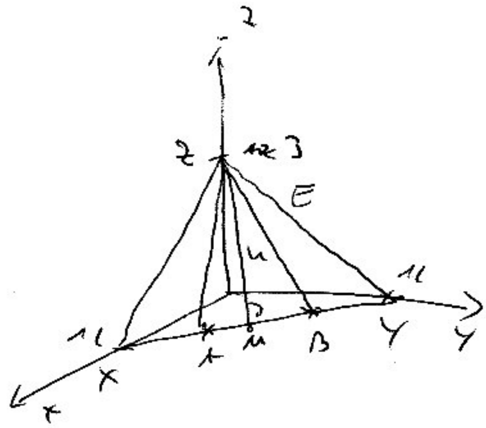
a) $E: \vec{x} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + s \cdot \underbrace{\begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix}}_{\overrightarrow{ZA}}$

$\vec{n}_E = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -8 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$

$E: \quad x + y + 4z + d = 0$

$Z(0|0|3): \quad d = -12$

$\underline{E: \quad x + y + z - 12 = 0}$



b) Grundriss (xy): $\vec{n}_{xy} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\cos\alpha = \dfrac{\vec{n}_E \cdot \vec{n}_{xy}}{|\vec{n}_E||\vec{n}_{xy}|} = \dfrac{4}{\sqrt{18}} = \dfrac{2}{3}\sqrt{2}$

$\underline{\alpha = 19{,}47^\circ}$

c) Schnitt mit x, y-Achse:

x-Achse ($y = z = 0$) $\quad x = 12 \quad \underline{X(12|0|0)}$

y-Achse ($x = z = 0$) $\quad y = 12 \quad \underline{Y(0|12|0)}$

d) $A_{ABZ} = 36\sqrt{2}$

$\vec{r}_M = \dfrac{\vec{r}_X + \vec{r}_Y}{2} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} \quad \overrightarrow{MZ} = \begin{pmatrix} -6 \\ -6 \\ 3 \end{pmatrix} \quad |\overrightarrow{MZ}| = \underline{h = 9}$

$A = \frac{1}{2} g \cdot h \Rightarrow \underline{g} = \dfrac{2A}{h} = \dfrac{2 \cdot 36\sqrt{2}}{9} = \underline{\mathbf{8\sqrt{2}}}$

$\overrightarrow{XY} = \begin{pmatrix} -12 \\ 12 \\ 0 \end{pmatrix}$; $|\overrightarrow{XY}| = 12\sqrt{2} = \frac{12}{8} \cdot g = \frac{3}{2} g$

$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{XY} \Rightarrow \overrightarrow{MB} = \frac{1}{3}\overrightarrow{XY} = \frac{1}{3} \cdot \begin{pmatrix} -12 \\ 12 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} \quad \underline{B = (2|10|0)}$

$\overrightarrow{MA} = -\overrightarrow{MB} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix} \quad \underline{A = (10|2|0)}$

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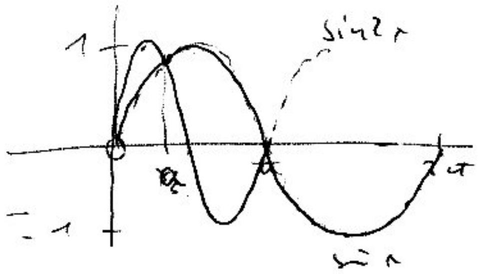
5.1. $f = \sin x$; $g = \sin 2x$

a) $f' = \cos x$; $g' = 2\cos 2x$



$f = g$

$\sin x = 2\sin x \cos x$

$\underbrace{\sin x}_{\substack{0 \\ \pi} + z \cdot 2\pi}(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} = a$ $+ z \cdot 2\pi$

$x = \frac{5}{3}\pi$

$x_S = a = \frac{\pi}{3}$

$f'(\frac{\pi}{3}) = \frac{1}{2} = \tan\alpha_1 \Rightarrow \alpha_1 = 26{,}57°$

$g'(\frac{\pi}{3}) = -1 = \tan\alpha_2 \Rightarrow \alpha_2 = -45°$

$\alpha = 71{,}6°$

b) $A = \int_0^a (g - f)\,dx = \left[-\frac{1}{2}\cos 2x - \cos x\right]_0^{\frac{\pi}{3}} = \frac{1}{4}$

5.2. $\vec{a} = \begin{pmatrix} x+1 \\ x-3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} -10 \\ 2x-7 \end{pmatrix}$

a) $\vec{a} = \vec{b}$?

I $x + 1 = -10$

II $x - 3 = 2x - 7$

keine Lösung, also nicht möglich $\vec{a} = \vec{b}$

b) $\vec{a} \perp \vec{b}$?

$\vec{a} \cdot \vec{b} = 0$

$(x+1)(-10) + (x-3)(2x-7) = 0$

$2x^2 - 23x + 11 = 0$

$x_1 = 11$

$x_2 = \frac{1}{2}$

zwei Möglichkeiten für $\vec{a} \perp \vec{b}$

c) $|\vec{a}| = |\vec{b}|$?

$(x+1)^2 + (x-3)^2 = 100 + (2x-7)^2$

$-2x^2 + 24x - 139 = 0$

$D < 0$, also nicht möglich $a = b$