Matur GF | F18

1. $f(x) = \frac{3x}{x^2-1}$

a) $f'(x) = 3 \cdot \frac{1(x^2-1) - x \cdot 2x}{(x^2-1)^2} = -3\,\frac{x^2+1}{(x^2-1)^2}$

$f''(x) = -3\,\frac{2x(x^2-1)^2 - (x^2+1)2(x^2-1)2x}{(x^2-1)^4} = 6\,\frac{x(x^2+3)}{(x^2-1)^3}$ ✓

b) NST: $x = 0$, einfach, VZW

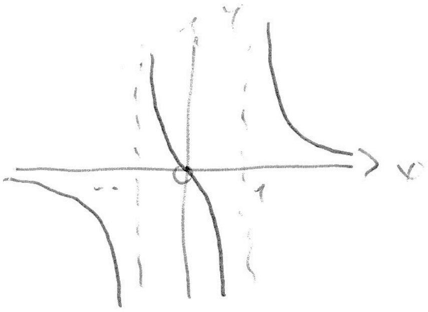
Ext.: $x^2+1 = 0$ (f)

WP: $x = 0$, einfach VZW, $W(0|0)$

Senk. A: $x = \pm 1$ VZW $D = \mathbb{R} \setminus \{\pm 1\}$

waag. A: $y = 0$ ($Z < N$)

P.S. $f(-x) = -f(x)$



c) $F(x) = \frac{3}{2} \ln(x^2-1)$

$F'(x) = \frac{3}{2} \cdot \frac{1}{x^2-1} \cdot 2x = \frac{3x}{x^2-1} = f(x)$ ✓

d) $\int_2^a f(x)\,dx = 5$

$\frac{3}{2}\left[\ln(x^2-1)\right]_2^a = 5$

$\ln(a^2-1) - \ln 3 = \frac{10}{3}$

$a^2-1 = e^{10/3 + \ln 3} = 3e^{10/3}$

$a = (\pm)\sqrt{3e^{10/3}+1}$ $a > 2$

2. A: 40% B: 60%

E_A: 50% E_B: 30%

a) 0,4 A: 0,5 E, 0,5 $\bar{E}$ $0{,}4 \cdot 0{,}5$

0,6 B: 0,3 E, 0,7 $\bar{E}$ $0{,}6 \cdot 0{,}7$ } 62%

b) $p = 0{,}1$ $\left(P(1 \text{ v. } 3) = \binom{3}{1} p^1 (1-p)^2 = 3 \cdot 0{,}1 \cdot 0{,}9^2 = 24{,}3\% \right)$

$P(1 \text{ v. } 3) = \frac{1}{10} + \frac{9}{10} \cdot \frac{1}{9} + \frac{9}{10} \cdot \frac{8}{9} \cdot \frac{1}{8} = 0{,}3 = 30\%$

c) $P(m|A) = \frac{0{,}4 \cdot 0{,}5}{0{,}4 \cdot 0{,}5 + 0{,}6 \cdot 0{,}7} = \frac{20\%}{62\%} = 32{,}3\%$

d) $P(N_{p=62\%}^{\text{mind.}}) > 99{,}99\%$

$1 - P(\text{kein}) > 0{,}9999$

$1 - (0{,}38)^N > 0{,}9999$

$N > \log_{0{,}38} \cancel{0{,}9999}\ 0{,}0001 = 9{,}51$

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3. $A(1|2|2)$ $B(3|4|3)$ $C(2|6|5)$

a) $D(0|4|4)$

$\vec{AB} = \begin{pmatrix}2\\2\\1\end{pmatrix}$ $\vec{DC} = \begin{pmatrix}2\\2\\1\end{pmatrix}$

$\vec{AB} = \vec{DC} \Rightarrow$ Parallelogramm

b) $\vec{AC} = \begin{pmatrix}1\\4\\3\end{pmatrix}$ $\cos\varphi = \frac{\vec{AC}\cdot\vec{DB}}{|\vec{AC}|\,|\vec{DB}|} = \frac{0}{\dots} = 0$

$\vec{DB} = \begin{pmatrix}3\\0\\-1\end{pmatrix}$

$\varphi = 90°$

c) $M = (\vec{r}_A + \vec{r}_C)/2 = \begin{pmatrix}3/2\\4\\7/2\end{pmatrix}$ $M(1,5|4|3,5)$

d) $E: \vec{x} = \vec{A} + s\vec{AB} + t\vec{AC} = \begin{pmatrix}1\\2\\2\end{pmatrix} + s\begin{pmatrix}2\\2\\1\end{pmatrix} + t\begin{pmatrix}1\\4\\3\end{pmatrix}$

e) $F(3,y,z)$ $\vec{AB}\times\vec{AC} = \begin{pmatrix}2\\-5\\6\end{pmatrix} \sim \vec{AF} = \begin{pmatrix}2\\y-2\\z-2\end{pmatrix}$ $\vec{n} = k\cdot\vec{AF}$

$= \vec{n}$ $k = 1$

$-5=y-2$

$y=-3$

$6=z-2$

$z=8$

f) $A = \frac{1}{2}ef = \frac{1}{2}\vec{AC}\cdot\vec{DB} = \frac{1}{2}\sqrt{26}\sqrt{10} = \sqrt{65}$

4 a) $f(x) = ax^3+bx^2+cx+3$; $f(3)=0$; $f'(3)=3$; $f''(3)=0$

$f(0)$ $27a+9b+3c+3=0$ | $27a+6b+c=3$ | $18a+2b=0$

$f(x) = -\frac{4}{9}x^3+4x^2-9x+3$

b) $x^2-4x+y^2-8y+11=0$ $P(4,4|2,2)$ $P \in k$: $0=0$ ✓

$M(2|4)$ $\vec{MP} = \begin{pmatrix}2,4\\-1,8\end{pmatrix}$ $t: \vec{x} = \begin{pmatrix}4,4\\2,2\end{pmatrix} + t\begin{pmatrix}1,8\\2,4\end{pmatrix}$

c) $y = e^{2x+2}$ $y = 2x$ min. Abstand $\Rightarrow \parallel$: $2e^{2x+2} = 2$

$e^{2x+2} = 1$

$x = -1$

d) $y = -\frac{1}{2}x^2+8$ NST: $x = \pm 4$

$A = \frac{8+2x}{2}\cdot f(x) = (4+x)\cdot(-\frac{1}{2}x^2+8)$

$A' = -\frac{3}{2}x^2-4x+8 = 0$

$x=-4$ | $x = \frac{4}{3}$ $A''(\frac{4}{3}) > 0 \Rightarrow$ Min $A(0)=0$ $A(4)=0$ $P(\frac{4}{3}|$ 64/9 $)$

e) $\int_0^{\frac{\pi}{4}} (\sin(2x+\pi)\,dx = [-\frac{1}{2}\cos(2x+\pi)]_0^{\frac{\pi}{4}} = -\frac{1}{2}$