Mat 6F F15

1. $f(x) = -x^3 + x$

$g(x) = x^2 + ax$

a) $a = -1$: $g(x) = x^2 - x$

NST: $\left.\begin{matrix} x = 0 \\ x = 1 \end{matrix}\right\}$ Scheitel $x = \frac{1}{2}$ $y = -\frac{1}{4}$ Min da P. nach oben geöff.

$f(x) = -x^3 + x = 0$

$\underbrace{x}(\underbrace{-x^2+1}) = 0$

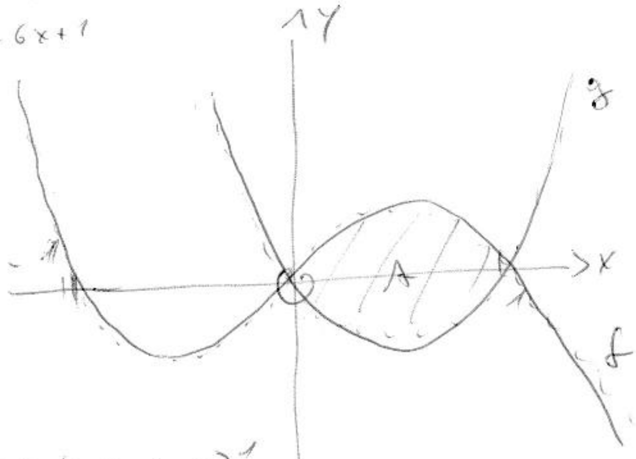
$\underline{x = 0}$ $\underline{x = \pm 1}$

$f'(x) = -3x^2 + 1$

$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{3}\sqrt{3}$ $f''(x_{1,2}) \gtrless 0$ Max$(\frac{1}{3}\sqrt{3} \mid \frac{2}{9}\sqrt{3})$ Min$(-\frac{1}{3}\sqrt{3} \mid -\frac{2}{9}\sqrt{3})$

$y = \pm\frac{2}{9}\sqrt{3}$

$f''(x) = -6x + 1$



$$\underline{A} = \int_0^1 (f-g)\,dx = \left[-\frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2\right]_0^1$$
$$= \underline{\frac{5}{12}}$$

b) g: $x - 2y - 14 = 0$

$y = \frac{1}{2}x - 7 = f(x)$

$2x^3 - x - 14 = 0$

$\pm1, \pm2, \pm7, \pm14$

$\underline{x = 2}$

$(2x^3 - x - 14) : (x-2) = 2x^2 + 4x + 7$ $D < 0$ $\underline{x = 2}$ einzige Lsg.

$f'(2) = -11$

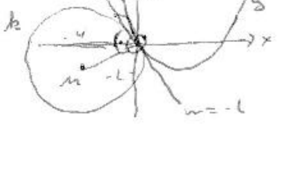
$\tan\alpha = \left|\frac{-11 - \frac{1}{2}}{1 - 11 \cdot \frac{1}{2}}\right|$

$\underline{\alpha = 111{,}4°}$

c) $y = x^2 + ax$

$y' = 2x + a$ $M(-4 \mid -2)$ $\vec{MO} = \binom{4}{2}$ $m = \frac{1}{2} \xrightarrow{\perp} m = -2 = y'(0)$

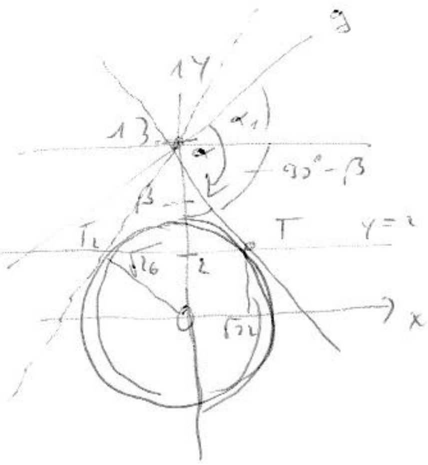
$y'(0) = a$ $\underline{-2 = a}$



Mat GF F15

2. $k: x^2 + y^2 = 26$

$Z(0|13)$



a) $g: y = x + 13$

$0 \cdot x + 13 \cdot y = 26$

$y = 2$ Polare

$x^2 + 4 = 26$

$x = \pm\sqrt{22}$ $\quad \underline{T_1(\sqrt{22}|2)} \quad T_2(-\sqrt{22}|2)$

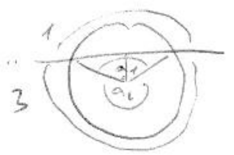
$\tan\beta = \frac{\sqrt{22}}{11} \Rightarrow \beta = 23{,}09°$ $\qquad \tan\alpha_1 = 1$

$\alpha_1 = 45°$

$\alpha = \alpha_1 + 90° - \beta$

$\underline{\alpha = 111{,}9°}$ $\qquad \underline{\alpha^* = \alpha + 2\beta}$

b) $y = c$



$b \sim \alpha$ d.h. $\frac{\alpha_1}{\alpha_2} = \frac{1}{3} \Rightarrow \alpha_1 = 90°$

$y = R \cdot \cos\left(\frac{\alpha_1}{2}\right) = \sqrt{26} \cdot \cos 45°$

$= \sqrt{26} \cdot \frac{1}{2}\sqrt{2}$

$\underline{y = \sqrt{13}}$

c) $y = -5x^2$

$x^2 + (5x^2)^2 = 26$

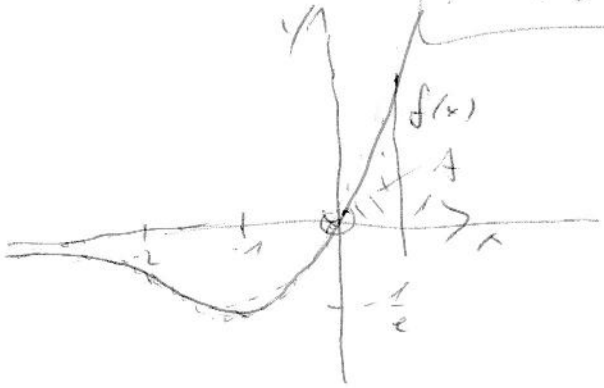
$26x^2 = 26$

$\underline{x = \pm 1}$ $\qquad \underline{S_1(1|-5)}$

$\underline{y = -5}$ $\qquad \underline{S_2(-1|-5)}$

Mat GF F15

3, $f(x) = x e^x$



a) $f'(x) = e^x + x e^x = (1+x)e^x$

$f''(x) = e^x + (1+x)e^x = (2+x)e^x$

$f'''(x) = e^x + (2+x)e^x = (3+x)e^x$

$\lim\limits_{x\to+\infty} x e^x = +\infty$

$\lim\limits_{x\to-\infty} x e^x = 0$ weil Exp. stärker als Potenz waag. A. x-Achse

NST $\underbrace{x \cdot e^x = 0}$
$\underline{x=0}$

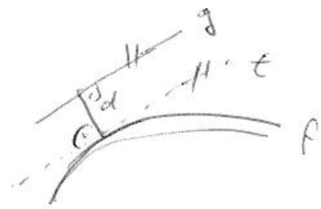
Ext. $\underbrace{(1+x)}\underbrace{e^x}_{\neq 0} = 0$
$\underline{x=-1}$
$\underline{y=-\frac{1}{e}}$

WP: $\underbrace{(2+x)}\underbrace{e^x}_{\neq 0} = 0$
$x=-2$
$y=-\frac{2}{e^2}$

$f''(-1) > 0 \Rightarrow$ Min $(-1|-\frac{1}{e})$

NST einfach, VZW, WP$(-2|-\frac{2}{e^2})$

b) g: $x - y - 13 = 0$
$y = x - 13$ $m = 1 \Rightarrow$ Kleinster Abstand $f'(x) = 1$
$f'(0) = 1$ ✓



also $P(0|0)$
ist Punkt mit kleinstem Abstand zu g

c) $F(x) = (x+k)e^x$

$F'(x) = e^x + (x+k)e^x = (\underbrace{1+k}+x)e^x$

$f(x) = (0 + x)e^x$

$\underline{\underline{k=-1}}$ $F(x) = (x-1)e^x$

$\underline{A} = \int_0^1 f(x)dx = [F(x)]_0^1 = (1-1)e^1 - (0-1)e^0 = \underline{1}$

(MatGF F15

4. $p(\text{defekt}) = \frac{1}{500} = 0{,}002$ $\quad q = 1 - p = 0{,}998$

a) $P(\text{20 intakt in 20}) = q^{20} = \underline{96{,}1\,\%}$

b) $P(\text{1 def. in 60}) = 60 \cdot p^1 \cdot q^{59} = \underline{10{,}7\,\%}$

c) $P(\text{mind. 2 defekt})$

$= 1 - P(\leq 1\,d.)$

$= 1 - (P(1\,d.) + P(0\,d.))$

$= 1 - 20 \cdot p^1 \cdot q^{19} - q^{20}$

$= \underline{0{,}0742\,\%}$

d) $P(\text{mind. 1 d. in n Lampen}) > 0{,}4$

$1 - P(\text{kein d. in n L.}) > 0{,}4$

$1 - q^n > 0{,}4$

$q^n < 0{,}6$

$n > \log_q 0{,}6 = 255{,}15$

<u>ab 256 Lampen</u>

$E(0|0|z)$

$\vec{AB}\,\vec{AB} = AC\cdot AB\cos\alpha$

$$\begin{pmatrix}0-2\\0-0\\z-0\end{pmatrix}\begin{pmatrix}0-2\\1-0\\0-0\end{pmatrix} = \sqrt{4+z^2}\cdot\sqrt{4+1^2}\cdot\cos 60°$$

$$4 = \sqrt{4+z^2}\cdot\sqrt{5}\cdot\frac{1}{2}$$

$$z = \pm\sqrt{\frac{44}{5}} = \pm\frac{2}{5}\sqrt{55}$$

b) $P(0|-1)$ Max in $f(x) = \dfrac{x^2+ax+b}{x-3}$

$$f'(x) = \frac{x^2-6x-3a-b}{(x-3)^2}$$

I $f(0) = -1$ I. $-\frac{b}{3} = -1 \Rightarrow b = 3$

II $f'(0) = 0$ II. $-\frac{1}{9}(3a+b) = 0 \rightarrow a = -1$

c) $f(x) = a\sin(bx)$ $a,b>0$

$x_{n+1} - x_n = \frac{\pi}{4}$

$\sin(bx=0)$

$bx_1 = 0 + z\cdot 2\pi$

$x_1 = 0 + z\cdot\frac{2\pi}{b}$

$bx_2 = \pi + z\cdot 2\pi$

$x_2 = \frac{\pi}{b} + z\cdot\frac{2\pi}{4}$

$x_{n+1} - x_n = \frac{\pi}{b} \overset{!}{=} \frac{\pi}{4}$

$b = 4$

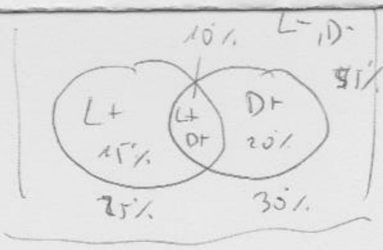
$f'(x) = a\cdot b\cdot\cos(bx)$

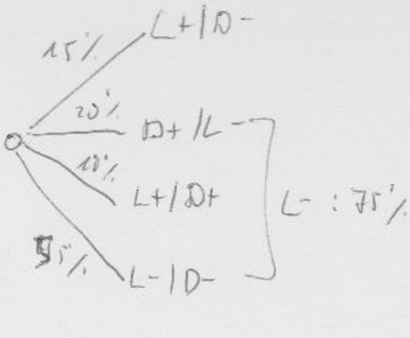
$f'(0) = ab = \tan(60°)$

$a\cdot 4 = \sqrt{3}$

$a = \frac{1}{4}\sqrt{3}$

d)





$\dfrac{D+|L-}{L-} = \dfrac{20\%}{75\%} = 26{,}7\%$

$(L+,D+)$: $25\% + 30\% - 10\% = 45\%$

$L-D-$: $1 - 45\% = 55\%$