Matur Winter 2012

1. a) $f(x) = -x^4 + x^2 + 12$ $\Rightarrow$ f ist achsensymmetrisch zur y-Achse

$f'(x) = -4x^3 + 2x$

$f''(x) = -12x^2 + 2$

$f'''(x) = -24x$

Nullstellen:

$-x^4 + x^2 + 12 = 0$

$x^4 - x^2 - 12 = 0$

$(x^2-4)(x^2+3) = 0 \rightarrow x^2 - 4 = 0$

$\underline{\underline{x_1 = -2}},\ \underline{\underline{x_2 = 2}}$

$x^2 + 3 = 0$

keine weitere Lösungen

Extremwerte

$f'(x) = -4x^3 + 2x = 0$

$-2x(2x^2-1) = 0 \Rightarrow x_3 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{\sqrt{2}}{2} \Rightarrow x_4 = -\frac{\sqrt{2}}{2} \quad x_5 = \frac{\sqrt{2}}{2}$

$f''(0) = 2 > 0 \rightarrow$ $\underline{\underline{\text{Tiefpunkt } T(0/12)}}$

$f''\left(\pm\frac{\sqrt{2}}{2}\right) = -12\cdot\frac{1}{2} + 2 = -4 < 0$

$\Rightarrow$ Hochpunkte $H_1\left(\frac{\sqrt{2}}{2}/12,25\right)$

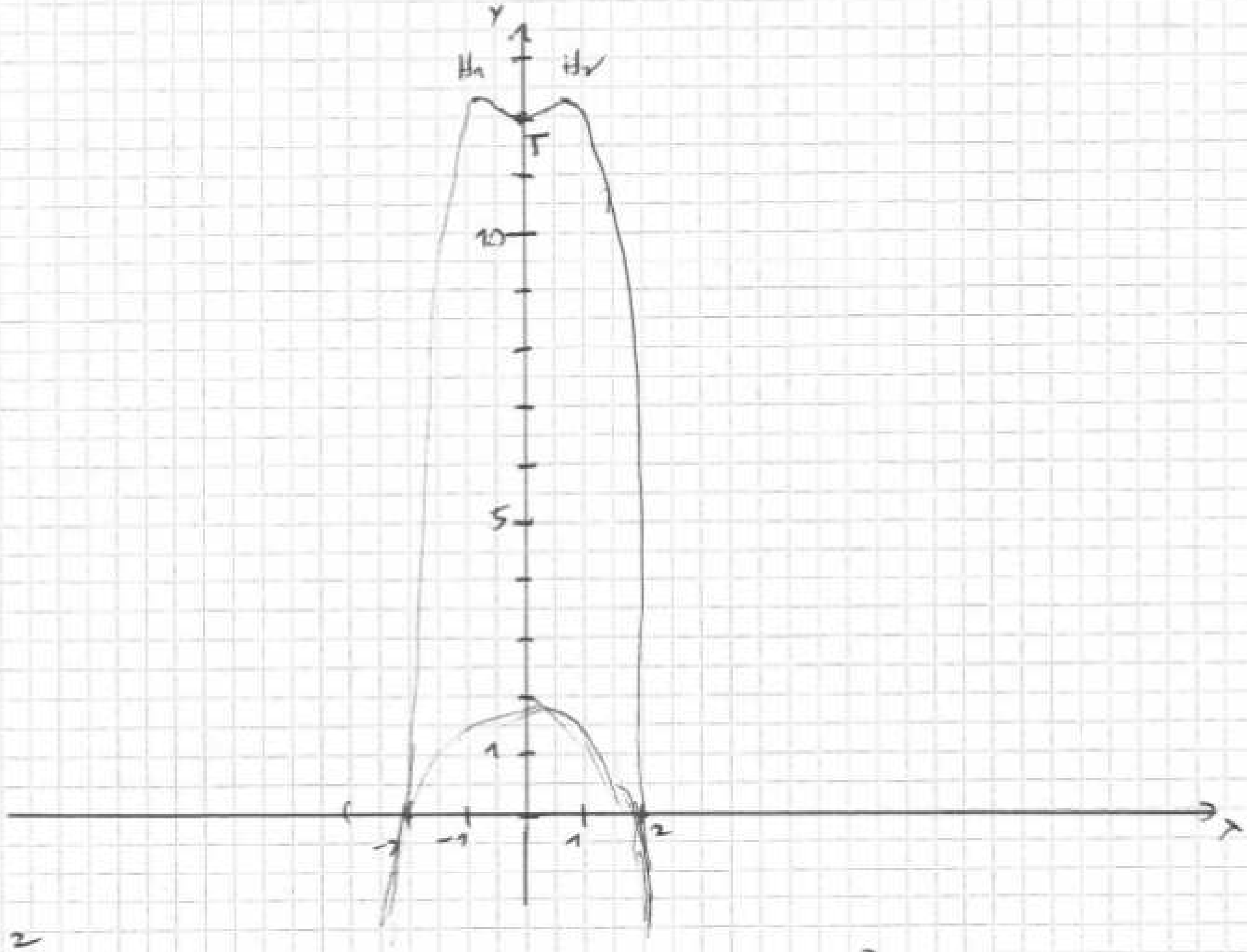
Wendepunkte: $-12x^2 + 2 = 0$

$12x^2 = 2$

$x^2 = \frac{1}{6} \Rightarrow x_6 = -\frac{\sqrt{6}}{6} \quad x_7 = \frac{\sqrt{6}}{6}$

$f'''\left(\pm\frac{\sqrt{6}}{6}\right) = \mp 4\sqrt{6} \neq 0$

$\rightarrow$ Wendepunkte $\underline{\underline{W_1\left(-\frac{\sqrt{6}}{6}/12,14\right)}}$ $\underline{\underline{W_2(0,41/12,14)}}$



$$A = \int_{-2}^{2} -x^4 + x^2 + 12 = \left[-\tfrac{1}{5}x^5 + \tfrac{1}{3}x^3 + 12 \right]_{-2}^{2}$$

$$= \left(+\tfrac{304}{15}\right) - \left(-\tfrac{304}{15}\right) = \underline{\underline{\tfrac{608}{15} = 40{,}5\overline{3}}}$$

b) Ansatz: $g(x) = y = ax^2 + bx + c$

$g'(x) = 2ax + b$

Bedingungen: • $g(-2) = g(2) = 0$

$$\left| \begin{array}{l} 4a + 2b + c = 0 \\ 4a - 2b + c = 0 \end{array} \right|$$

$4b = 0 \Rightarrow b = 0$

$4a + c = 0$

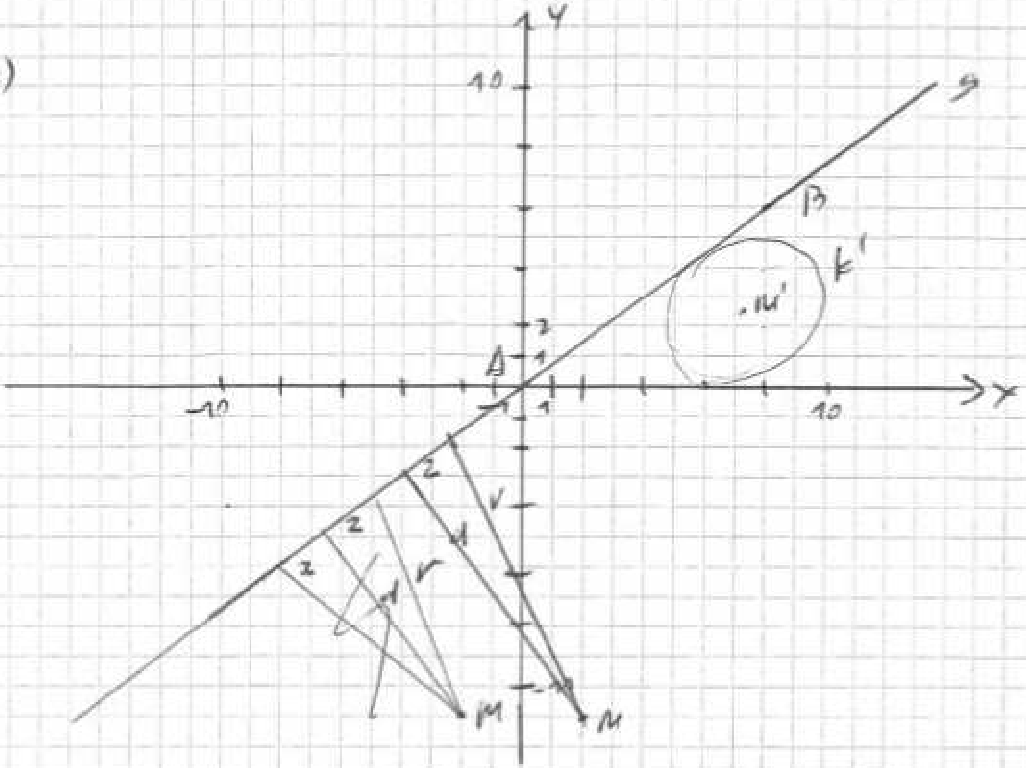
• $g'(2) = f'(2) = -28$:

$4a = -28 \Rightarrow a = -7$

$\Rightarrow c = -4a = -28$

$\underline{\underline{g(x):\ y = -7x^2 + 28}}$

2. a)



$g = AC$: $\quad \underline{m = \frac{6-0}{8-0} = \frac{3}{4}} \quad \rightarrow \quad g: \; y = \frac{3}{4}x$

$4y = 3x$

$g: \; 3x - 4y = 0$

$HNF_g: \quad \frac{3x-4y}{5} = 0$

$d = \frac{3 \cdot 2 - 4 \cdot (-11)}{5} = 10$

$r^2 = d^2 + 2^2 = 104 \quad \Rightarrow \quad \underline{\underline{k: \; (x-2)^2 + (y+11)^2 = 104}}$

b) Ansatz: $M'(u/v) \qquad v = r = 2$

M' in HNF_g einsetzen:

$$\left| \frac{3u - 4 \cdot 2}{5} \right| = 2$$

$\frac{3u-8}{5} = 2 \qquad$ oder $\qquad \frac{3u-8}{5} = -2$

$3u - 8 = 10 \qquad\qquad 3u - 8 = -10$

$3u = 18 \qquad\qquad 3u = -2$

$u = 6 \qquad\qquad u = -\frac{2}{3}$ → Berührt x-Achse im negativen Bereich!

$\rightarrow \underline{\underline{M'(6/2)}}$

3. a) $f(x) = \frac{8x-8}{x^2} = \frac{8}{x} - \frac{8}{x^2}$

$f'(x) = -\frac{8}{x^2} + \frac{16}{x^3}$

$f''(x) = \frac{16}{x^3} - \frac{48}{x^4}$

Nullstellen: $8x - 8 = 0 \Leftrightarrow 8x = 8 \Leftrightarrow \underline{x_1 = 1}$

Extremalpunkte: $-\frac{8}{x^2} + \frac{16}{x^3} = 0 \quad | \cdot x^3$

$-8x + 16 = 0$

$8x = 16 \qquad x_2 = 2$

$f''(2) = \frac{16}{8} - \frac{48}{16} = 2 - 3 = -1 < 0$

Hochpunkt $H(2/2)$

$f'(-2) = -2 + \frac{16}{(-8)} = -4$

$\tan(90° - \varphi) = |-4| = 4$

$90° - \varphi = 75,96°$

$\varphi = 14,04°$

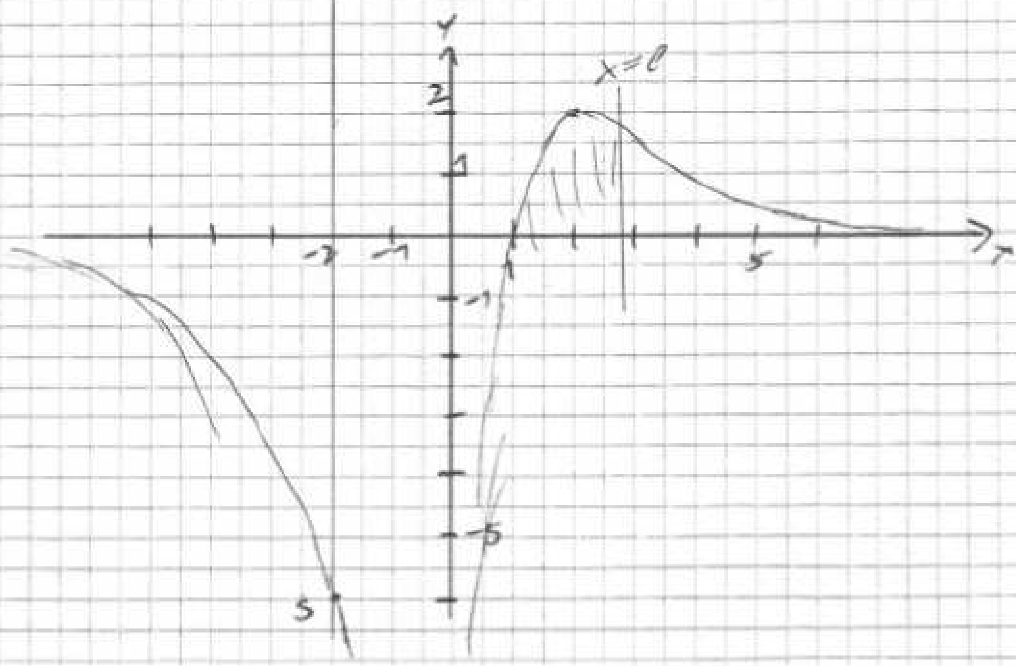
$f(-2) = -6$

Schnittpunktes mit Grade $x = -2$: $S(-2/-6)$

Asymptotisches Verhalten: senkrechte Asymptote $x = 0$

Grad Zähler < Grad Nenner

x-Achse ist waagrechte Asymptote



b) $\int_1^e \left(\frac{8}{x} - \frac{8}{x^2}\right) dx = \left[8 \cdot \ln|x| + \frac{8}{x}\right]_1^e$

$= \left(8 \cdot \ln e + \frac{8}{e}\right) - \left(8 \cdot \ln 1 + \frac{8}{1}\right)$

$= 8 + \frac{8}{e} - 0 - 8 = \underline{\underline{\frac{8}{e} = 2{,}943}}$

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4. a) $A(-2|-2) \quad B(2|0)$

$\overline{AB} = \sqrt{(2-(-2))^2 + (0-(-2))^2} = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

$\underline{\underline{h}} = \frac{\sqrt{3}}{2} \cdot 2\sqrt{5} = \underline{\underline{\sqrt{15} = 3{,}873}}$

$\sqrt{20} = \overline{CA} = \overline{CB} \iff \overline{CA}^2 = \overline{CB}^2 = 20$ mit $C(x|y)$

$$\left| \begin{array}{l} (-2-x)^2 + (-2-y)^2 = 20 \\ (2-x)^2 + (0-y)^2 = 20 \end{array} \right|$$

$$- \left| \begin{array}{l} 4 + 4x + x^2 + 4 + 4y + y^2 = 20 \\ 4 - 4x + x^2 \qquad\qquad + y^2 = 20 \end{array} \right|$$

$8x + 4 - 4y = 0$

$4y = 8x + 4$

$\underline{y = 2x - 1}$

$(2-x)^2 + (1+2x)^2 = 20$

$4 - 4x + x^2 + 1 + 4x + 4x^2 = 20$

$5x^2 \cancel{- 8x} - 15 = 0$

$5x^2 = 15$

$x^2 = 3$

$x_1 = \sqrt{3} \;,\; x_2 = -\sqrt{3}$

$y_1 = -2\sqrt{3} - 1 \quad y_2 = 2\sqrt{3} - 1$

$\underline{\underline{C_1(\sqrt{3}|-2\sqrt{3}-1) \qquad C_2(-\sqrt{3}/2\sqrt{3}-1)}}$

b) $C(x/y)$ $g: x-2y+1=0 \Rightarrow x=2y-1$

$C \in g$: $x-2y+1=0$

$\overrightarrow{CA} \perp \overrightarrow{CB}$: $\begin{pmatrix}-2-x\\-2-y\end{pmatrix} \cdot \begin{pmatrix}2-x\\0-y\end{pmatrix} = 0$

$(-2-x)(2-x) + (-2-y)\cdot(-y) = 0$

$\left| \begin{matrix} -4+x^2+2y+y^2=0 \\ x = 2y-1 \end{matrix} \right|$

$(2y-1)^2 - 4 + 2y + y^2 = 0$

$1-4y+4y^2-4+2y+y^2=0$

$5y^2-2y-3=0$

$D = 4+4\cdot5\cdot3 = 64 \Rightarrow \sqrt{D} = 8$

$y_1 = \frac{2+8}{10} = 1$ $y_2 = \frac{2-8}{10} = -\frac{3}{5}$

$x_1 = 1$ $x_2 = -\frac{11}{5}$

$\underline{\underline{C_1(1/1)}}$ $\underline{\underline{C_2 = (-\frac{11}{5}/-\frac{3}{5})}}$

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5. a) $\underline{\underline{P}} = \left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^5\cdot\left(\frac{1}{2}\right)^1 = 7\cdot\left(\frac{1}{2}\right)^6 = \underline{\underline{\frac{7}{64}}}$

b) $\underline{\underline{P}} = \left(\frac{1}{2}\cdot\frac{2}{5}\right)^2 = \underline{\underline{\frac{1}{25}}}$ ~~$(=P($~~

c) $\underline{\underline{P}} = \binom{12}{3}\cdot\left(\frac{1}{5}\right)^3\cdot\left(\frac{4}{5}\right)^9 = \underline{\underline{0{,}236}}$

d) $1-\left(\frac{4}{5}\right)^n > 0{,}99$

$\left(\frac{4}{5}\right)^n < 0{,}01$

$n\cdot\ln 0{,}8 < \ln 0{,}01$

$n > \frac{\ln 0{,}01}{\ln 0{,}8} = 20{,}64$

<u>Mind. 21 Blüten</u>

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c) $A(p|0|0)$, $B(0|3p|0)$, $C(0|0|2p)$ $\quad p>0$

$$\overrightarrow{AB} = \begin{pmatrix} -p \\ 3p \\ 0 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} -p \\ 0 \\ 2p \end{pmatrix}$$

$$\cos\varphi = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\overline{AB} \cdot \overline{AC}} = \frac{(-p)\cdot(-p)}{\sqrt{10}\,p \cdot \sqrt{5}\,p} = \frac{1}{\sqrt{50}}$$

$$\underline{\underline{\varphi = 81{,}87^\circ}}$$

d) $f(x) = 2x^3 - bx - 4$

$f(2) = 2\cdot 8 - 2b - 4 = 0$

$12 = 2b$

$b = 6$

$f(x) = 2x^3 - 6x - 4$

$(2x^3 - 6x - 4) : (x-2) = 2x^2 + 4x + 2$

$-(2x^3 - 4x^2)$

$4x^2 - 6x - 4$

$-(4x^2 - 8x)$

$2x - 4$

$-(2x - 4)$

0

$2(x^2 + 2x + 1) = 0$

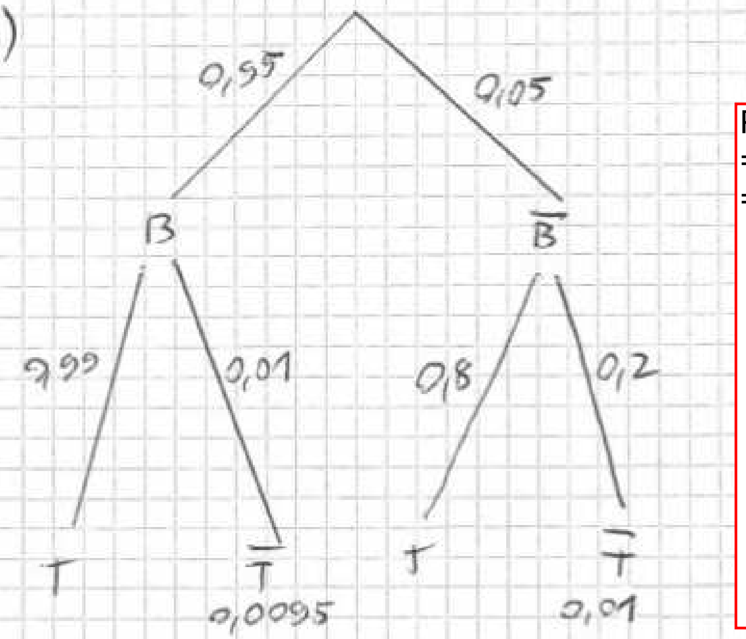
$(x+1)^2 = 0$

$\underline{\underline{x_2 = -1}}$

e)

B: Bild ok.

T: Ton ok.



P(Bild ok, wenn Ton defekt)
= 0,0095 / 0,0195
= 48,7%

$P(\overline{T}) = 0{,}95 \cdot 0{,}01 + 0{,}05 \cdot 0{,}2 = 0{,}0195$

6. a) Graph von f ist eine Sinuskurve mit

Amplitude $a=2$, nach zwei Einheiten nach oben

verschoben: $y_{max}=4$ q.e.d

Variante: Maximaler Funktionswert bei z.B. $x=\frac{\pi}{6}$

$$f\left(\frac{\pi}{6}\right)= 2\cdot\sin\frac{\pi}{2}+2=4$$

$$f(x)=2\sin(3x)+2=3$$

$$2\sin(3x)=1$$

$$\sin(3x)=\frac{1}{2}$$

$$3x=\frac{\pi}{6}$$

$$x=\frac{\pi}{18}$$

$$f'(x)= 2\cos(3x)\cdot 3=6\cos(3x)$$

$$f'\left(\frac{\pi}{18}\right)=6\cos\frac{\pi}{6}=6\cdot\frac{\sqrt{3}}{2}=3\sqrt{3}$$

$$\tan\varphi=3\sqrt{3}$$

$$\underline{\underline{\varphi=79{,}1^\circ}}$$

b) $g(x)=2e^{2x}-1$

Nullstellen: $2e^{2x}-1=0$

$$2e^{2x}=1$$

$$e^{2x}=\frac{1}{2}$$

$$2x=\ln\frac{1}{2}=-\ln 2$$

$$x=-\frac{\ln 2}{2}$$

$$\int_{-\frac{\ln 2}{2}}^{0}(2e^{2x}-1)\,dx=\left[e^{2x}-x\right]_{-\frac{\ln 2}{2}}^{0}=(1-0)-\left(\frac{1}{2}+\frac{\ln 2}{2}\right)$$

$$=\frac{1}{2}-\frac{\ln 2}{2}=\underline{\underline{\frac{1}{2}(1-\ln 2)}}=0{,}153$$