1) $\boxed{f(x) = x^3 - ax^2}$, $f'(x) = 3x^2 - 2ax$, $f''(x) = 6x - 2a$, für $a > 0$

Nullstellen: $0 = x^2(x-a)$ $x_1 = 0$, $x_2 = a$ $\underline{N_1(0|0)}$, $\underline{N_2(a|0)}$

Extrema: $0 = x(3x - 2a)$ $x_1 = 0$, $x_2 = \frac{2}{3}a$

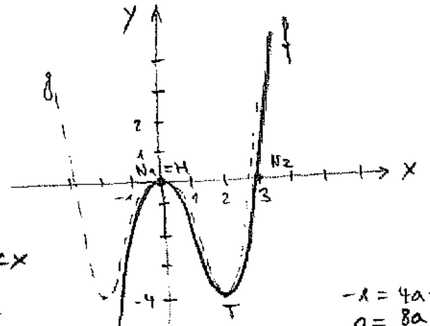
$f''(0) = -2a$ → Max. weil $a > 0$

$f''(\frac{2}{3}a) = 2a$ → Min. weil $a > 0$

$f(0) = 0$

$f(\frac{2}{3}a) = \frac{4}{9}a^2(\frac{2}{3}a - \frac{3}{3}a) = -\frac{4}{27}a^3$ $\underline{H = N_1(0|0)}$, $\underline{T(\frac{2}{3}a | -\frac{4}{27}a^3)}$

a) $\underline{a = 3}$ $\underline{\underline{N_1 = H(0|0)}}$, $\underline{\underline{N_2(3|0)}}$, $\underline{\underline{T(2|-4)}}$



Bedingungen: $g(x) = ax^4 + bx^3 + cx^2 + dx + e$

▷ Achsensymmetrie: $g(x) = ax^4 + cx^2 + e$

▷ y-Achsenabschnitt bei $N_1 = H(0|0)$, d.h. $e = 0$: $\boxed{g(x) = ax^4 + cx^2}$, $g'(x) = 4ax^3 + 2cx$

▷ Extrema bei $T(2|-4)$: $g(2) = 16a + 4c$ $g'(2) = 32a + 4c$

$\underline{-4 = 16a + 4c}$ $\underline{0 = 32a + 4c}$

$-1 = 4a + c$ $| \cdot(-1) \cdot (-2)$

$\underline{0 = 8a + c}$

$\underline{1 = 4a}$ $\underline{a = \frac{1}{4}}$

$\underline{2 = -c}$ $\underline{c = -2}$

$\underline{\underline{g(x) = \frac{1}{4}x^4 - 2x^2}}$

b) $\boxed{t: y - y_P = m(x - x_P)}$

$m = f'(x) = 3x^2 - 2ax$, $N_2(a|0)$ $y = a^2(x - a)$, $P(0|-8)$

$f'(a) = 3a^2 - 2a^2$ $-8 = -a^3$

$f'(a) = a^2$ $\underline{\underline{2 = a}}$ $\underline{t: y = 4x - 8}$

2) $f(x) = \frac{2x^2}{(x+5)(x-2)}$, $f'(x) = \frac{2x(3x-20)}{(x+5)^2(x-2)^2}$, $f''(x) = \frac{-4(3x^3 - 30x^2 - 100)}{(x+5)^3(x-2)^3}$

a) ▷ Pol 1. Ordnung bei $\underline{\underline{x = -5}}$ } weil $\mathbb{D} = \mathbb{R} \setminus \{-5, 2\}$
Pol 1. Ordnung bei $\underline{\underline{x = 2}}$

▷ $\underline{m = n}$

$y = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{10}{x^2}} = \lim_{x\to\infty} \frac{2}{1 + \frac{3}{x} - \frac{10}{x^2}} = 2$ $\underline{\underline{y = 2}}$ ⇒ Die Aussage stimmt, f hat Asymptoten bei $\underline{x = -5}$, $\underline{x = 2}$ und $\underline{y = 2}$

b) $0 = 2x(3x - 20)$ $\underline{x_1 = 0}$ $\underline{x_2 = \frac{20}{3}}$ → mögl. Extr. stellen

$f''(0) = \frac{400}{5^3(-2)^3} = -\frac{400}{5^3 2^3} \underline{< 0 \to \text{Max.}}$

$f''(\frac{20}{3}) = \frac{-4(\frac{20^3}{9} - \frac{30 \cdot 20^2}{9} - \frac{9 \cdot 10^2}{9})}{(\frac{20}{3} + \frac{15}{3})^3 (\frac{20}{3} - \frac{6}{3})^3} = \frac{-\frac{4}{9}(2^3 \cdot 10^3 - 3 \cdot 2^2 \cdot 10^3 - 9 \cdot 10^2)}{\frac{35^3}{3^3} \cdot \frac{14^3}{3^3}}$

$= \frac{-4 \cdot 3 \cdot 10^2(80 - 120 - 9)}{35^3 \cdot 14^3} = \frac{4 \cdot 3 \cdot 10^2 \cdot 49}{35^3 \cdot 14^3} \underline{> 0 \to \text{Min}}$

⇒ Die Aussage ist falsch
f hat 2 Extrema: $\underline{H_T(0|0)}$, $\underline{H_H(\frac{20}{3}|\frac{80}{49})}$

c) $y = -\frac{1}{2}x$ $-\frac{1}{2}x = \frac{2x^2}{x^2 + 3x - 10}$

$x^3 + 3x^2 - 10x = -4x^2$

$x^3 + 7x^2 - 10x = 0$

$x(x^2 + 7x - 10) = 0$ $\underline{x_1 = 0}$, $D = \sqrt{49 + 40}$ $D > 0$, d.h. 2 Lösungen x_2, x_3

⇒ Die Aussage ist richtig, g und f schneiden sich in 3 Punkten.

[illegible] Niveau

3) a) $g: \quad y = 2x$

$n: \quad y+6 = -\frac{1}{2}(x-7)$

$y = -\frac{1}{2}x + \frac{7}{2} - \frac{12}{2}$

$n: \quad y = -\frac{1}{2}x - \frac{5}{2}$

$g \cap n = \{S\}$

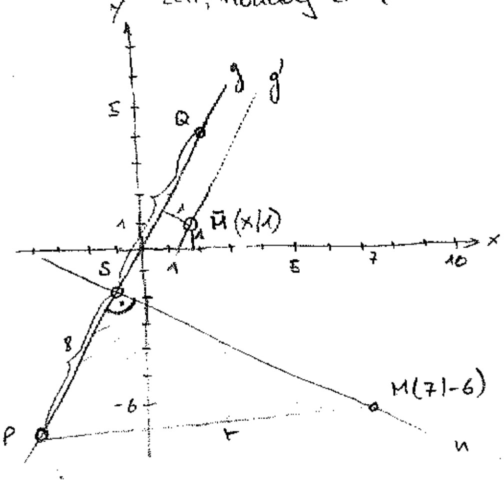
$2x = -\frac{1}{2}x - \frac{5}{2}$

$4x = -x - 5$

$5x = -5$

$x = -1$

$S = (-1|-2)$



$|\vec{SM}| = |\vec{r}_M - \vec{r}_S| = \left|\begin{pmatrix}7\\-6\end{pmatrix} - \begin{pmatrix}-1\\-2\end{pmatrix}\right| = \sqrt{64+16} = \sqrt{80}$

$r^2 = \overline{SM}^2 + \overline{PS}^2$

$r^2 = 80 + 64$

$r = 12$

$k: (x-7)^2 + (y+6)^2 = 144$

$g: \quad y = 2x$

$k \cap g = \{P, Q\}$

$(x-7)^2 + (2x+6)^2 = 144$

$x^2 - 14x + 49 + 4x^2 + 24x + 36 = 144$

$5x^2 + 10x - 59 = 0$

$x_{1,2} = \frac{-10 \pm \sqrt{100 + 4 \cdot 5 \cdot 59}}{10} = \frac{-10 \pm \sqrt{1280}}{10}$

$x_{1,2} = \frac{-10 \pm 16\sqrt{5}}{10} = -1 \pm \frac{8}{5}\sqrt{5}$

$P\left(-\frac{8}{5}\sqrt{5} - 1 \,\middle|\, -\frac{16}{5}\sqrt{5} - 2\right)$, $Q\left(\frac{8}{5}\sqrt{5} - 1 \,\middle|\, \frac{16}{5}\sqrt{5} - 2\right)$

b) ▸ $g' \parallel g$ mit Abstand $d = 1$, wobei $g: 2x - y = 0$

$1 = \pm\left|\frac{2x-y}{\sqrt{4+1}}\right|$

$\sqrt{5} = 2x - y$ $\qquad$ $\sqrt{5} = -2x + y$

$y = 2x - \sqrt{5}$ $\qquad$ $2x + \sqrt{5} = y$

$\Rightarrow \quad g': \; y = 2x - \sqrt{5}$

▸ $\bar{H}(x|1)$

$1 = 2x - \sqrt{5} \quad \Leftrightarrow \quad x = \frac{1}{2}(\sqrt{5}+1)$

$\bar{H}\left(\frac{1}{2}(\sqrt{5}+1) \,\middle|\, 1\right)$

4) a) Urne mit 10 roten, 13 schwarzen Kugeln ⇒ 23

4 mal hintereinander ziehen ohne zurücklegen → Baumdiagramm

$\frac{10}{23}$ r — $\frac{9}{22}$ — r — $\frac{13}{21}$ — s — $\frac{12}{20}$ — s; r — s ....; $\frac{13}{23}$ s — r ...., s ... (unvollständig)

$P(rrss) = \frac{10}{23} \cdot \frac{9}{22} \cdot \frac{13}{21} \cdot \frac{12}{20} = \frac{9 \cdot 10 \cdot 12 \cdot 13}{20 \cdot 21 \cdot 22 \cdot 23} = \frac{9 \cdot 13}{7 \cdot 11 \cdot 23}$

Anzahl Möglichkeiten: (rrss), (rsrs), (rssr), (srrs), (srsr), (ssrr) ⇒ 6

$P(2s) = 6 \cdot \frac{9 \cdot 13}{7 \cdot 11 \cdot 23} \approx 0{,}396$

b) ▸ 3 mal hintereinander würfeln und 3 gerade Zahlen würfeln (g = gerade Zahl)

$P(g) \cdot P(g) \cdot P(g) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

▸ Anzahl Würf mit 3 Würfeln = n

$\left(\frac{1}{8}\right)^n < 10^{-16}$

$2^{-3n} < 10^{-16}$

$\lg 2^{-3n} < \lg 10^{-16}$

$-3n \lg 2 < -16 \lg 10$

$n > \frac{-16}{-3\lg 2} \quad \Rightarrow \quad n > \frac{16}{3\lg 2}$ $\qquad$ 18 Dreierwürf

c) 2 mal hintereinander ziehen ohne zurücklegen.

gelbe Kugeln: x

blaue Kugeln: 100 − x

$\frac{x}{100}$ g — $\frac{x-1}{99}$ — g; g — $\frac{100-x}{99}$ — b; b

$P(gg) = \frac{x}{100} \cdot \frac{x-1}{99}$

$P(gb, bg) = \frac{x}{100} \cdot \frac{100-x}{99} + \frac{100-x}{100} \cdot \frac{x}{99}$

$P(gg) \leq P(gb, bg)$

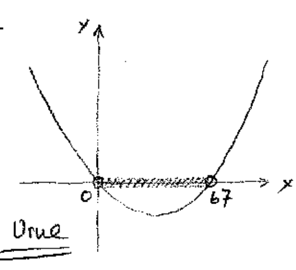
$\frac{x(x-1)}{99 \cdot 100} \leq \frac{x(100-x) + x(100-x)}{99 \cdot 100}$

$x(x-1) \leq 2x(100-x)$

$x^2 - x \leq 200x - 2x^2$

$3x^2 - 201x \leq 0$

$x(x-67) \leq 0$



Es hat höchstens 67 Kugeln in der Urne

5) a) $A = \frac{1}{2} |\vec{a} \times \vec{b}|$

$A = \frac{1}{2} \left| \begin{pmatrix} -2 \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ \frac{2}{b} \end{pmatrix} \right| = \frac{1}{2} \left| \begin{pmatrix} 2 \\ \frac{4}{b} \\ 2b \end{pmatrix} \right|$

$\sqrt{6} = \frac{1}{2} \sqrt{4 + \frac{16}{b^2} + 4b^2}$

$6 = \frac{1}{4} \cdot 4 \left(1 + \frac{4}{b^2} + b^2\right)$

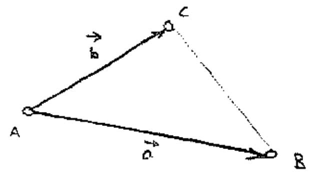
$5b^2 = 4 + b^4$

$0 = b^4 - 5b^2 + 4$

$0 = (b^2-4)(b^2-1)$

$0 = (b+2)(b-2)(b+1)(b-1)$

$\mathbb{L} = \{-2, -1, 1, 2\}$



b) $g: \; y = -x + 3 \qquad \vec{r} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\vec{b} = t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\vec{a} - \vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - t \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3-t \\ 4+t \end{pmatrix}$

$\vec{a} + \vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3+t \\ 4-t \end{pmatrix}$

$(\vec{a} - \vec{b}) \perp (\vec{a} + \vec{b}): \; (\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = \cos 90°$

$\begin{pmatrix} 3-t \\ 4+t \end{pmatrix} \cdot \begin{pmatrix} 3+t \\ 4-t \end{pmatrix} = 0$

$9 - t^2 + 16 - t^2 = 0$

$25 - 2t^2 = 0$

$t = \pm \frac{5}{\sqrt{2}} \qquad t = \pm \frac{5}{2}\sqrt{2} \qquad \vec{b}_1 = \frac{5}{2}\sqrt{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

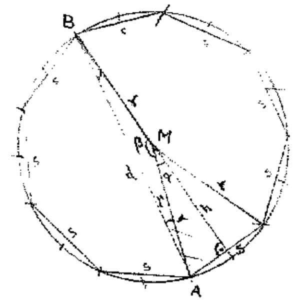
$\vec{b}_2 = -\frac{5}{2}\sqrt{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

6) a) $\alpha = \frac{360°}{2n} \qquad n = \text{Anzahl Ecken} \qquad \alpha = 20°$

$\sin\alpha = \frac{\frac{s}{2}}{r} \Leftrightarrow r = \frac{s}{2\sin\alpha} \Leftrightarrow r = \frac{s}{2\sin 20°}$

Δ ABM ist gleichschenklig, wobei gilt: $\beta = 180° - \alpha$ (Nebenwinkel), $\beta = 160°$

$\gamma = \frac{1}{2}(180° - \beta) \quad , \; \gamma = 10°$



$\frac{\sin\gamma}{r} = \frac{\sin\beta}{d} \Leftrightarrow d = \frac{r \cdot \sin\beta}{\sin\gamma}$

$d = \frac{s \cdot \sin 160°}{2\sin 20° \cdot \sin 10°} \qquad | \sin 160° = \sin 20°$

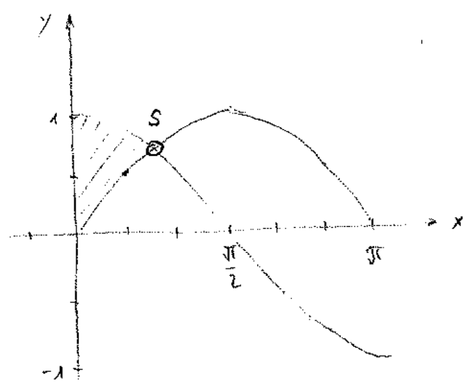
$d = \frac{s}{2\sin 10°} \qquad d \approx 2{,}88s$

b) $f(x) = \sin x \qquad \frac{1}{2} = \sin x \qquad x = \frac{\pi}{6} \qquad S\left(\frac{\pi}{6} / \frac{1}{2}\right)$

$g(x) = k\cos x \qquad \frac{1}{2} = k\cos\frac{\pi}{6}$

$\frac{1}{2} = k\frac{\sqrt{3}}{2}$

$k = \frac{\sqrt{3}}{3}$



$A = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}}{3}\cos x - \sin x \; dx$

$A = \left[\frac{\sqrt{3}}{3}\sin x + \cos x\right]_0^{\frac{\pi}{6}}$

$A = \frac{\sqrt{3}}{3}\sin\frac{\pi}{6} + \cos\frac{\pi}{6} - \frac{\sqrt{3}}{3}\sin 0 - \cos 0 = \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{2} - 0 - 1 = \frac{1}{6}(\sqrt{3} + 3\sqrt{3} - 6) = \frac{1}{6}(4\sqrt{3} - 6) = \frac{2}{3}\sqrt{3} - 1$