EN H15

1. $f(x) = \frac{ax^2}{x-a}$; $a > 0$

a) $D = \mathbb{R} \setminus \{a\}$ Senk. A. $x = a$

NST.: $x = 0$ doppelt (Extr.)

$x^2 : (x-a) = x + a + \frac{a^2}{x-a}$ $\rightarrow$ $a\cdot(x+a)$ schräge A. $(= g(x))$

$\frac{x^2 - ax}{\quad}$
ax
$\frac{ax - a^2}{\quad}$
a^2

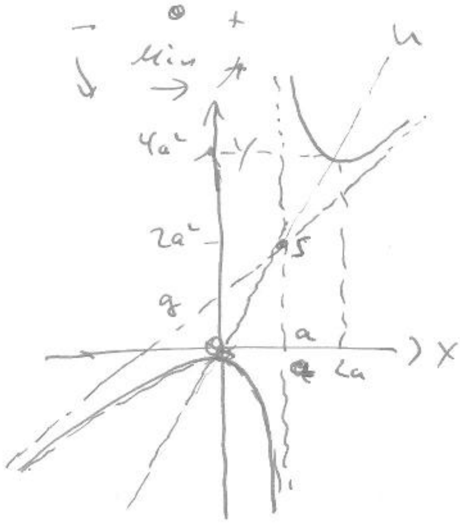
$f'(x) = a \cdot \frac{2x(x-a) - x^2}{(x-a)^2} = a \frac{2x^2 - 2ax - x^2}{(x-a)^2} = a \frac{x^2 - 2ax}{(x-a)^2}$

Extr. $x = 0$ $y = 0$
$x = 2a$ $y = 4a^2$

x	$-a$	0	$\frac{1}{2}a$	$\frac{3}{2}a$	$2a$	$3a$
$f'(x)$	$\frac{3}{4}a$	0	$-3a$	$-3a$	0	$\frac{3}{4}a$
	>0					
	$+$	0	$-$	$-$	0	$+$
f	$\nearrow$	Max	$\searrow$	$\searrow$	Min	$\nearrow$

Max $(0|0)$
Min $(2a|4a^2)$

$W =]-\infty; 0] \cup [4a^2; \infty[$



b) s. o.

c)

d) $h(x)$: $m = \frac{4a^2}{2a} = 2a$

$h(x) = 2a \cdot x$

e) $2ax = ax + a^2$
$ax = a^2$
$x = a$ $y = 2a^2$ $S(a|2a^2)$

$\tan\alpha = \left| \frac{2a - a}{1 + 2a \cdot a} \right| = \frac{a}{1+2a^2}$

$a = 1$: $\tan\alpha = \frac{1}{3}$
$\alpha = 18{,}43°$

f) $\alpha \rightarrow \max$ d.h. $\tan\alpha \rightarrow \max$
$\tan\alpha$ streng monoton steigend

d.h. $\frac{a}{1+2a^2} \rightarrow \max$

$\frac{1 + 2a^2 - a\,4a}{(1+2a^2)^2} = \frac{1-2a^2}{(1+2a^2)^2} = 0$

$a = \pm\frac{1}{2}\sqrt{2}$

$a = \frac{1}{2}\sqrt{2}$

EN 415

2a) Quadrat : 1 $\qquad \frac{3-\sqrt{2}}{7} \simeq 0{,}227$

Dreieck : $\frac{1}{2}$ $\qquad \frac{3-\sqrt{2}}{14} \simeq 0{,}113$

Rechteck : $\sqrt{2}$ $\qquad \frac{3\sqrt{2}-2}{7} \simeq 0{,}320$

$p(ABC) = p(DEF) \approx 0{,}1$ △

$p(ABDE) = p(BCEF) = 0{,}25$ □

$p(ACDE) = 0{,}3$ ▭

b) $P(\text{alle } \triangle) = 0{,}1^3 = \underline{0{,}1\,\%}$

c) $P(\text{alle verschieden}) = 6 \cdot 0{,}1 \cdot 0{,}25 \cdot 0{,}3 = \underline{4{,}5\,\%}$

d) $P(n, \text{ mind. ein aus } \triangle) > 0{,}95$

$1 - P(\text{kein } \triangle) > 0{,}95$

$1 - 0{,}9^n > 0{,}95$

$0{,}9^n < 0{,}05$

$n > \log_{0{,}9} 0{,}05 = 28{,}4$

$\underline{n \geq 29}$

e) $P(▭ \mid □) = \frac{0{,}3}{0{,}3 + 2 \cdot 0{,}25} = \underline{37{,}5\,\%}$

ENH1Γ

3. a) $\det M = \left(\frac{3}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = -4 \neq 0 \Rightarrow$ invertierbar

$M = M^T$, also ~~orthogonal~~ symmetrisch

aber $M^T M \neq 1$, also nicht orthogonal, Bildvektoren können gestreckt und nicht nur gedreht/gespiegelt sein

b) $M\vec{v} = \lambda \vec{v}$

$M\vec{v} - \lambda\vec{v} = 0$

$(M - \lambda 1)\vec{v} = 0$

$\begin{pmatrix} \frac{3}{2}-\lambda & \frac{5}{2} \\ \frac{5}{2} & \frac{3}{2}-\lambda \end{pmatrix} \vec{v} = 0 \quad | \det$

$\left(\frac{3}{2}-\lambda\right)^2 - \left(\frac{5}{2}\right)^2 = 0$

$\underline{\lambda_1 = -1}$
$\underline{\lambda_2 = 4}$

$\left(\frac{3}{2}+1\right)x + \frac{5}{2}y = 0$

$x = -y$

$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \underline{\vec{v}_{1,0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$

$\left(\frac{3}{2}-4\right)x + \frac{5}{2}y = 0$

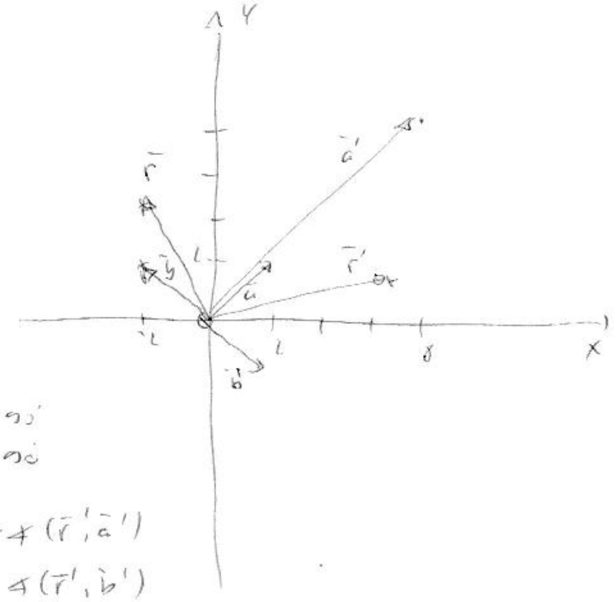
$x = y$

$\vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \underline{\vec{v}_{2,0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

c) $M \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} = \vec{a}'$

$M \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \vec{b}'$

$M \begin{pmatrix} 1-3 \\ 1+3 \end{pmatrix} = M \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} = \vec{r}'$



$\vec{a}, \vec{b}$ Eigenvektoren zu $-1, 4$

also ist $\vec{a}' = 4\vec{a}$ $\quad \sphericalangle(\vec{a},\vec{b}) = 90°$
$\vec{b}' = -\vec{b}$ $\quad \sphericalangle(\vec{a}',\vec{b}') = 90°$

$\vec{r}' = \frac{1}{2}\vec{a}' + \frac{3}{2}\vec{b}'$ $\quad \sphericalangle(\vec{r},\vec{a}) \neq \sphericalangle(\vec{r}',\vec{a}')$
$\sphericalangle(\vec{r},\vec{b}) \neq \sphericalangle(\vec{r}',\vec{b}')$

4. $E(6|2|1)$ $F(2|6|3)$ $A(6|5|4)$ EN H15

a) $\overrightarrow{EF} = \begin{pmatrix} -4 \\ 4 \\ 2 \end{pmatrix}$ $\overrightarrow{EA} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$

$\vec{n} = \overrightarrow{EF} \times \overrightarrow{EA} = \begin{pmatrix} 6 \\ 12 \\ -12 \end{pmatrix}$: $x + 2y - 2z + d = 0$

$E: 6 + 4 - 2 + d = 0$

$\underline{E(AEF): x + 2y - 2z - 8 = 0}$

b) $A(AEF) = \frac{1}{2} |\overrightarrow{EF} \times \overrightarrow{EA}| = \frac{1}{2} 6 \cdot 3 = \underline{9}$

c) $K: M(0|2|7)$ $R = 6$

$d(M; E_{AEF}) = \left| \frac{0 + 4 - 14 - 8}{3} \right| = 6$ ✓

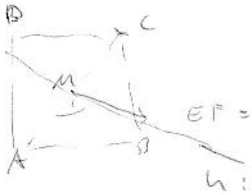
d $g(M; \vec{n}): \vec{x} = \begin{pmatrix} 0 \\ 2 \\ 7 \end{pmatrix} + r \cancel{\begin{pmatrix} 6 \\ 12 \\ -12 \end{pmatrix}} \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cap E_{EAF}$

$r = 2$

$T(2|6|3)$

d) 

$\underline{R'} = \sqrt{6^2 + 8^2} = \underline{10}$

e) 

$\overrightarrow{EF} = \begin{pmatrix} -4 \\ 4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$ $E_{Quad}: -2x + 2y + z + d = 0$

A $-12 + 10 + 4 + d = 0$

$h: \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} + s \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$

$E_Q: -2x + 2y + z - 2 = 0$

$E_Q \cap h: s = 1$

$\underline{M(4|4|2)}$

$\vec{r}_C = \vec{r}_A + 2\overrightarrow{AM} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ $\underline{C(2|3|0)}$ $\overrightarrow{AM} = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix}$ $\overline{AM} = 3$

$\cancel{\vec{r}_B}$ $\overrightarrow{EF} \times \overrightarrow{AM} = \begin{pmatrix} 6 \\ 12 \\ -12 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $|\,| = 3$

$\vec{r}_B = \vec{r}_M - (\quad " \quad) = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$ $\underline{B(3|2|4)}$

$\vec{r}_D = \vec{r}_M + (\quad " \quad) = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix}$ $\underline{D(5|6|0)}$

ENHA

5. a) $f_1(x) = 2\sin x + 1$

NST $\sin x = -\frac{1}{2}$

$x_1 = -\frac{\pi}{6} + z\cdot 2\pi \rightarrow \frac{11}{6}\pi$

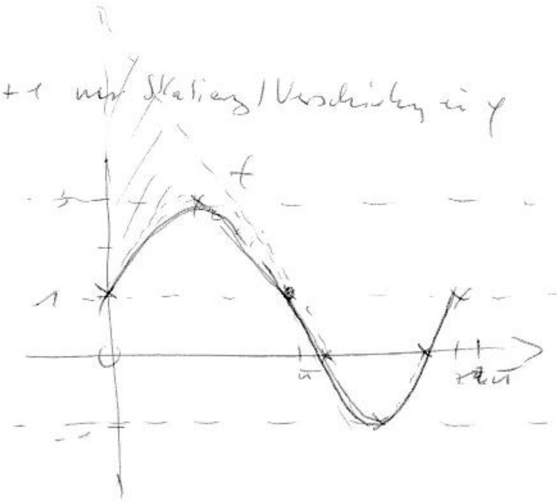
$x_2 = -\frac{5}{6}\pi + z\cdot 2\pi \rightarrow \frac{7}{6}\pi$

Ext. $f_1 \Leftrightarrow$ Ext. $\sin(x)$ da 2· und +1 nur Skalierung / Verschiebung in y

Max $\left(\frac{\pi}{2} + z\cdot 2\pi \,\middle|\, 3\right)$

Min $\left(\frac{3}{2}\pi + z\cdot 2\pi \,\middle|\, -1\right)$

$\mathbb{D} = \mathbb{R}$; $\mathbb{W} = [-1; 3]$



b) $f_1'(x) = 2\cos x$

$f_1'(\pi) = -2$

$t:\ y = -2(x-\pi) + 1$

$A = \int_0^{\pi} (t - f_1)\,dx = \left[-2\left(\frac{x^2}{2} - \pi\right) + \cancel{x} + 2\cos x - \cancel{x}\right]_0^{\pi}$

$= \underline{\pi^2 - 4}$

c) $f_2(x) = A\sin(bx+c) + \frac{1}{2}$ Periode $\pi \rightarrow \underline{b = 2}$

$= A\sin(2x+c) + \frac{1}{2}$

I $f_2(0) = f_1(0)$

II $f_2'(0) = f_1'(0)$ $f_2'(x) = 2A\cos(2x+c)$

I $A\sin c + \frac{1}{2} = 1 \Rightarrow A = \frac{1}{2\sin c}$

II $2A\cos c = 2$

$2\cdot\frac{1}{2\sin c}\cos c = 2$

$\tan c = \frac{1}{2}$

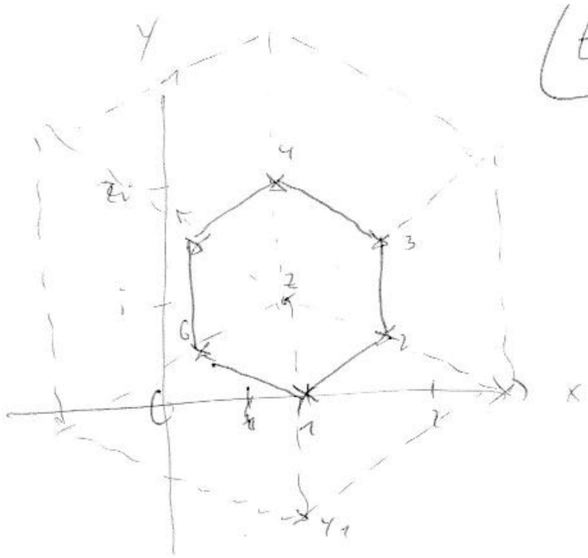
$c = \arctan\left(\frac{1}{2}\right)$ $A = \frac{\sqrt{5}}{2}$

$f_2(x) = \frac{\sqrt{5}}{2}\sin\left(2x + \arctan\left(\tfrac{1}{2}\right)\right) + \frac{1}{2} = \frac{\sqrt{5}}{2}\sin\left(2x + \arctan\left(\tfrac{1}{2}\right)\right) + \frac{1}{2}$

$\left(= \cos^2(x) + 2\sin x\cos x\right)$

(EN HA15

6.
$z_1 = 1$
$z_2 = \frac{1}{2}(2+\sqrt{3}) + \frac{1}{2}i$
$z_3 = \frac{1}{2}(2+\sqrt{3}) + \frac{3}{2}i$
$z_4 = 1 \quad 2i$
$z_5 = \frac{1}{2}(2-\sqrt{3}) + \frac{3}{2}i$
$z_6 = \frac{1}{2}(2-\sqrt{3}) + \frac{1}{2}i$



b) $z_2 - z_1 = \frac{1}{2}\sqrt{3} + \frac{1}{2}i = s$
$|s| = \frac{1}{2}\sqrt{3+1} = 1$ $\quad \underline{u = 6}$
$\underline{A} = 6 \cdot \frac{1^2}{4}\sqrt{3} = \underline{\frac{3}{2}\sqrt{3}}$

c) $z_{n+1} = a z_n + b \quad ; \; z_1 = 1$

I $\quad \frac{1}{2}(2+\sqrt{3}) + \frac{1}{2}i = a + b \quad \longrightarrow \quad b = \frac{1}{2}(2+\sqrt{3}) + \frac{1}{2}i - a$

II $\quad 1 + 2i = \left(\frac{1}{2}(2+\sqrt{3}) + \frac{3}{2}i\right) a + b$

$1 + 2i = \left(\frac{1}{2}(2+\sqrt{3}) + \frac{3}{2}i\right) a + \frac{1}{2}(2+\sqrt{3}) + \frac{1}{2}i - a$

$-\frac{1}{2}\sqrt{3} + \frac{3}{2}i = \left(1 + \frac{1}{2}\sqrt{3} + \frac{3}{2}i - 1\right) a$

$\underline{a} = \frac{-\frac{1}{2}\sqrt{3} + \frac{3}{2}i}{\frac{1}{2}\sqrt{3} + \frac{3}{2}i} = \underline{\frac{1}{2} + \frac{1}{2}\sqrt{3}i}$

$\underline{b = \frac{1+\sqrt{3}}{2} + \frac{1-\sqrt{3}}{2}i}$

d) $z = az + b$ $\qquad 1 + i \quad$ (Mitte 6-Eck)

$\underline{z} = \frac{b}{1-a} = \frac{\frac{1+\sqrt{3} + (1-\sqrt{3})i}{2}}{\frac{1 - \sqrt{3}i}{2}} = \frac{1+\sqrt{3} + (1-\sqrt{3})i}{1 - \sqrt{3}i} = \underline{\cancel{\frac{1-\sqrt{3}}{2} + \frac{1+\sqrt{3}}{2}i}}$

$z_1 = z_7 = 1 \overset{?}{=} a z_6 + b$

$z_6 = \frac{1-b}{a} = \frac{\frac{1-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2}i}{\frac{1+\sqrt{3}i}{2}} = \frac{1-\sqrt{3} + (\sqrt{3}-1)i}{1+\sqrt{3}i} = 1 - \frac{\sqrt{3}}{2} + \frac{1}{2}i = \underline{z_6}$ ✓

e) $y_{n+1} = a y_n + b \quad ; \; y_1 = 1 - i$

Ein 6-Eck mit Mitte $1+i$, Kantenlänge 2