LEN #13

1. a) $\vec{AC} \cdot \vec{CG} = \begin{pmatrix} 8 \\ 2 \\ -2 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix} = 0$ ✓

$$\left.\begin{array}{l} |\vec{AC}| = \sqrt{72} \\ |\vec{CG}| = \sqrt{36} \end{array}\right\} \quad AC = CG \cdot \sqrt{2} \quad ✓$$

b) $\vec{AC} \times \vec{CG} = \begin{pmatrix} 0 \\ 36 \\ 36 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$$x + y + z + d = 0$$

$C(10|1|1)$ : $\quad d = -12$

$\underline{E_{ACG}: \quad x + y + z - 12 = 0}$

c) $\vec{r}_E = \vec{r}_A + \vec{CG} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} \qquad \underline{E(0|3|-1)}$

Mitte AC : $\vec{r}_M = \frac{\vec{r}_A + \vec{r}_C}{2} = \begin{pmatrix} 6 \\ 0 \\ +2 \end{pmatrix}$

$\vec{r}_B = \vec{r}_M + \frac{1}{2}\frac{\sqrt{72}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 5 \end{pmatrix} \qquad \cancel{B(6|26|28)} \quad \underline{B(6|3|5)}$

d) $\vec{v} = t\begin{pmatrix} 1 \\ 2 \\ k \end{pmatrix} \qquad \vec{v} \cdot \vec{n} = t\begin{pmatrix} 1 \\ 2 \\ k \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = (2+k)t = t\sqrt{5+k^2}\sqrt{2} \cdot \frac{1}{2}\sqrt{2}$

$$(2+k)^2 = 5 + k^2$$

$$\underline{k = \frac{1}{4}}$$

$g: \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix} = \vec{x}$

in E: $\quad 4 + 4t + 1 + 8t + 1 + t - 12 = 0$

$$t = \frac{6}{13}$$

$$\underline{S\left(\frac{76}{13} \middle| \frac{61}{13} \middle| \frac{19}{13}\right)}$$

ENH13

2. a) i. $4^{16}$ $= 4{,}3$ Mrd.

ii. $16 \cdot 15 \cdot 14 \cdot 13$ $= 43680$

b) i. $\left(\frac{1}{4}\right)^{16}$

ii. $\frac{16!}{(4!)^4} \cdot \left(\frac{1}{4}\right)^{16} = 1{,}47\%$

c) 3 z. o. Z. aus 10

$P(\text{gleiche Farbe}) = \frac{5 \cdot 4 \cdot 3}{10 \cdot 9 \cdot 8} = \frac{1}{12}$

$P(\text{fortlaufend}) = \frac{8 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8} = \frac{1}{45}$

$P(\underset{4,5,6}{\text{g.F und fort.}}) = \frac{1}{10 \cdot 9 \cdot 8} = \frac{1}{720} = p_1$

Erhalt $= 2 \cdot (P(\text{g.F}) - p_1) + 4\,(P(\text{f.}) - p_1) + 30 \cdot p_1$

$= \frac{13}{45}$

Verlust von $\frac{32}{45}$ Fr $\approx$ 71 Rp. pro Spiel.

3. a) $f(x) = \dfrac{2x^2 - k}{x+1}$ , $k > 0, \neq 2$ EN 1113

$\mathbb{D} = \mathbb{R} \setminus \{-1\}$

NST: $\underline{x} = \pm\sqrt{\frac{k}{2}} = \underline{\pm \frac{1}{2}\sqrt{2k}}$

b) $f'(x) = \dfrac{2x^2 + 4x + k}{(x+1)^2}$

$f'(x) = 0$

$\underline{x_\pm = \dfrac{-2 \pm \sqrt{2(2-k)}}{2}}$ $\underline{k < 2}$

$f' = \frac{Z}{N}$ : $N > 0$, Quadrat

Z: Parabel, nach oben geöffnet, linke NST $x_-$
Vorzeichenwechsel von $+ \to -$ : Max
rechte NST $x_+$ " " $- \to +$ : Min

c) $k = 1{,}5$

$f(x) = \dfrac{2x^2 - 3/2}{x+1} = \underbrace{2x-2} + \dfrac{1}{2(x+1)}$

$\underline{y = 2x - 2}$ schräge A.

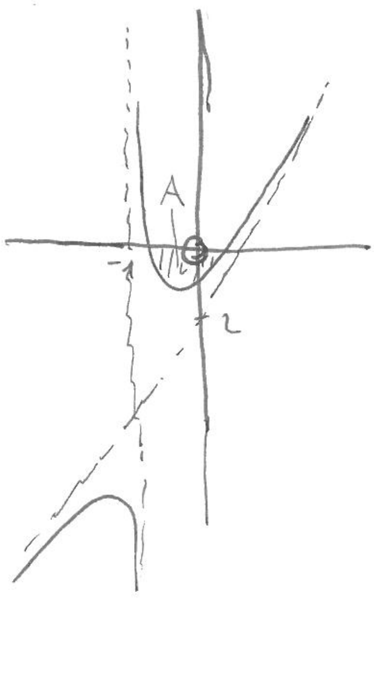
d) $2x^2 - \frac{3}{2} = 0$

$\underline{x = \pm \frac{1}{2}\sqrt{3}}$

$f'(x) = \dfrac{4x^2 + 8x + 3}{2(x+1)^2} = 0$

$\underline{x_1 = -\frac{3}{2} \quad y_1 = -6 \quad \text{Max}}$

$\underline{x_2 = -\frac{1}{2} \quad y_2 = -2 \quad \text{Min}}$




e) $\underline{A} = -\displaystyle\int_{-\frac{1}{2}\sqrt{3}}^{\frac{1}{2}\sqrt{3}} f(x)\,dx$

$= -\left[x^2 - 2x + \frac{1}{2}\ln(|x+1|)\right]_{-\frac{1}{2}\sqrt{3}}^{\frac{1}{2}\sqrt{3}}$

$= \underline{+2\sqrt{3} \mp \frac{1}{2}\ln\left(\frac{\sqrt{3}+2}{2-\sqrt{3}}\right)}$

$\approx 2{,}147$

LENHO

4.1. $f(z) = \left(\frac{1}{2} + k \cdot i\right) z$

a) $k = 1$ (i) $f_1(z) = \left(\frac{1}{2} + i\right) z$

$z_1 = 1$ ; $z_1 = 1 \cdot cis(0°)$

$z_2 = \frac{1}{2} + i$ ; $z_2 = \frac{1}{2}\sqrt{5}\ cis(63{,}4°)$

$z_3 = \left(\frac{1}{2}+i\right)\left(\frac{1}{2}+i\right) = -\frac{3}{4} + i$ ; $z_3 = \frac{1}{2}\sqrt{13} \cdot cis(143{,}1°)$

$\vdots$

$z_n = \left(\frac{1}{2}+i\right)^{n-1} = \left(\frac{1}{2}\sqrt{5}\right)^{n-1} \cdot cis(63{,}4° \cdot (n-1))$

$$\lim_{z \to \infty} \underbrace{\left(\frac{1}{2}\sqrt{5}\right)^{n-1}}_{\to \infty} \cdot cis(63{,}4°(n-1)) = \infty$$

b) $f_k(z) = \left(\frac{1}{2} + k i\right) z$

$z_1 = 1$

$z_2 = \left(\frac{1}{2} + k i\right)$

$z_3 = \left(\frac{1}{2} + k i\right)^2$

$\vdots$

$z_m = \left(\frac{1}{2} + k i\right)^{m-1} = 1$

$= \underbrace{\sqrt{k^2 + \frac{1}{4}}}_{1}{}^{m-1} \cdot cis\left(\frac{360}{m-1} \cdot \ell\right) = 1 \qquad \ell = 0; \ldots m-2$

$k = -\frac{1}{2}\sqrt{3}$ $k = \frac{1}{2}\sqrt{3}$

4.2. $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ENH13

$$\begin{vmatrix} 1-\lambda & 0 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$-(1-\lambda)(1+\lambda) - 0 = 0$$

$$\underline{\underline{\lambda = \pm 1}}$$

$+1:$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = +1 \cdot \begin{pmatrix} a \\ b \end{pmatrix}$

$$\left.\begin{matrix} a = a \\ a - b = b \end{matrix}\right\} \quad a = 2b \qquad \begin{pmatrix} 2b \\ b \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}}\, b$$

$-1:$ $\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -1 \cdot \begin{pmatrix} a \\ b \end{pmatrix}$

$$\left.\begin{matrix} a = -a \\ a - b = -b \end{matrix}\right\} \quad \begin{matrix} a = 0 \\ b \in \mathbb{R} \end{matrix} \qquad \begin{pmatrix} 0 \\ b \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}}\, b$$

4.3. $x^2 - 4y^2 + 8y = 0$

$x^2 - 4(y^2 - 2y) = 0$

$x^2 - 4(y^2 - 2y + 1 - 1) = 0$

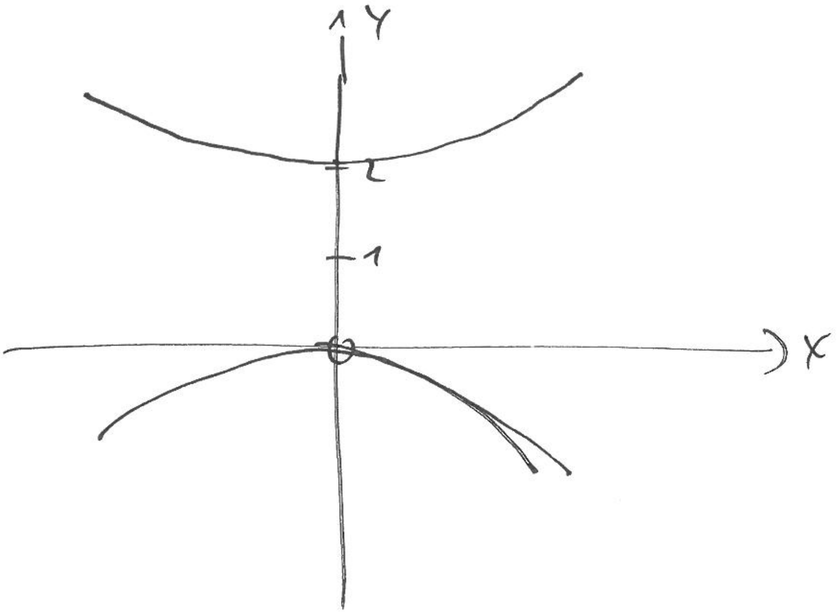
$x^2 - 4(y-1)^2 = -4$ Symmetriezentrum $\underline{(0|1)}$

$a = 1,\ b = 2\ ;\ c = \sqrt{5}$

Asymptoten: $\underline{y = 1 \pm \frac{1}{2}x}$

Scheitel: $\underline{(0|0)\ ;\ (0|2)}$

Brennpunkt: $\underline{(0|1 \pm \sqrt{5})}$

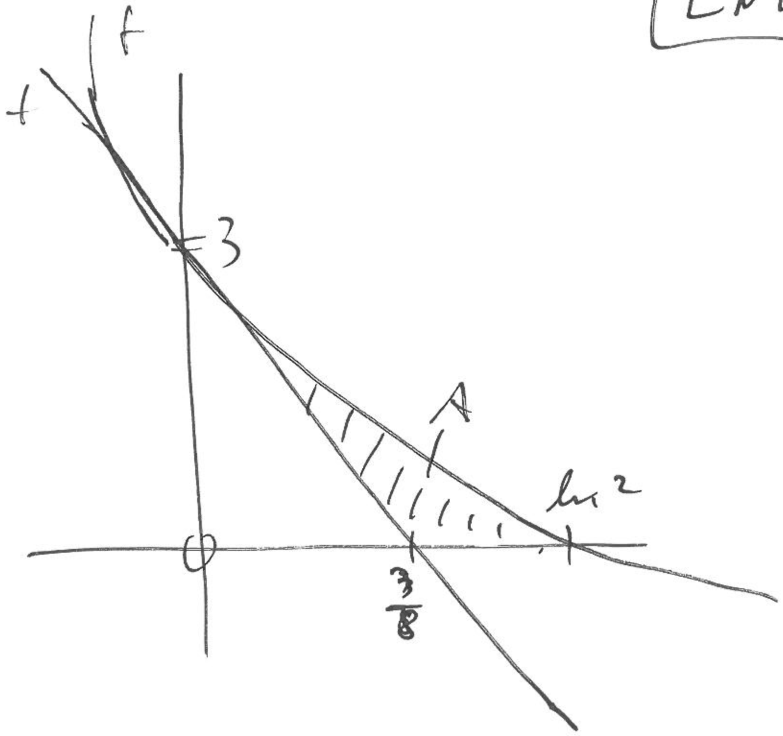


LEN H13

5.1. $f(x) = 4e^{-2x} - 1$

a) $f'(x) = -8e^{-2x}$

$t: \quad y = -8x + 3$




$f(x) = 0$

$e^{-2x} = \frac{1}{4}$

$-2x = \ln \frac{1}{4}$

$x = -\frac{1}{2} \ln 4^{-1}$

$x = \ln 2$

b)

$$V = \pi \int_0^{\ln 2} f^2(x)\,dx - V_{Kegel}\left(r = 3; h = \frac{3}{8}\right)$$

$$= \pi \int_0^{\ln 2} \left(16e^{-4x} - 8e^{-2x} + 1\right) dx - \frac{\pi}{3}\, 9 \cdot \frac{3}{8}$$

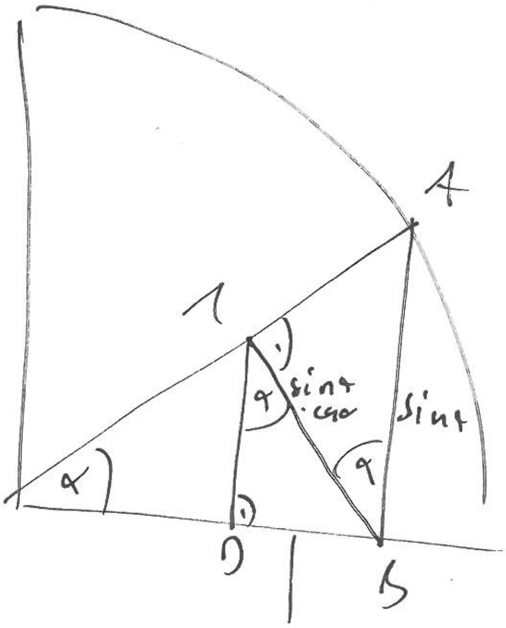
$$= \pi \left[-4e^{-4x} + 4e^{-2x} + x\right]_0^{\ln 2} - \frac{9}{8}\pi$$

$$= \pi\left(\ln 2 + \frac{3}{4}\right) - \frac{9}{8}\pi$$

$$\underline{\underline{V = \pi\left(\ln 2 - \frac{3}{8}\right)}}$$

5.2

ENH13




$$DB = \sin\alpha \cdot \cos\alpha \cdot \sin\alpha$$

$$DB = \sin^2\alpha \cos\alpha$$

$$DB' = 2\sin\alpha\cos\alpha \mp \sin^2\alpha\sin\alpha$$

$$= \sin\alpha\,(\underbrace{2\cos\alpha - \sin^2\alpha}) = 0$$

$\alpha = 0$, $= 180°$ | $\cos^2\alpha + 2\cos\alpha - 1 = 0$

$$\cos\alpha = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$$

$$\cos\alpha = -1 + \sqrt{2}$$

$$\underline{\underline{\alpha = 65{,}53°}}$$

Ränder: $DB(0°) = 0$, $DB(90°) = 0$ } Min

VzT

| $\alpha$ | 60° | 65,53 | 70° |
|---|---|---|---|
| $DB'$ | + | 0 | − |
| | | Max | |

LENMB

5.1. $f(x) = 4e^{-2x} - 1$

a) $f'(x) = -8e^{-2x}$

$\left.\begin{array}{l} f(0) = 4 - 1 = -3 \\ f'(0) = -8 \end{array}\right\}$ $t: \underline{y = -8x - 3}$

b) $f(x) = 0$

$\underline{x} = -\frac{1}{2}\ln\frac{1}{4} = \underline{\ln 2}$

$f(x) = 0$

$\underline{x = \frac{3}{8}}$

$$V = \pi\int_0^{\ln 2} f^2(x)\,dx - \underbrace{\pi\int_0^{\frac{3}{8}} f^2(x)\,dx}_{\text{Kegel } r = 3;\ h = \frac{3}{8}}$$

$$= \pi\int_0^{\ln 2}\left(1 - 8e^{-2x} + 16e^{-4x}\right)dx - \frac{27}{8}\pi$$

$$= \pi\left[x + 4e^{-2x} - 4e^{-4x}\right]_0^{\ln 2} - \frac{27}{8}\pi$$

$$= \pi\left(\ln 2 + \frac{3}{4}\right) - \frac{27}{8}\pi$$

$$\underline{V = \pi\left(\ln 2 - \frac{21}{8}\right)}$$