EN
H12

1.

a) $f'(x) = 6x - 6k = 0$

$\underline{x = k}$

$\underline{y = -k^2}$

Scheitel $(k|-k^2)$ einer nach oben offenen Parabel ist Minimum.

b) $k = -2k^2$

$2k^2 - k = 0$

$\underbrace{k}\underbrace{(2k-1)} = 0$

$(k=0)$ $\underline{k = \frac{1}{2}}$

c) $3(x-k)^2 - k^2 = 0$

$x - k = \pm\sqrt{\frac{k^2}{3}}$

$x_{1/2} = k \pm \frac{1}{3}\sqrt{3}\,k$

$f'(k + \frac{1}{3}\sqrt{3}k) = 6k + 2\sqrt{3}k - 6k = 2\sqrt{3}k = \tan\alpha$

$\underline{\alpha = \arctan(2\sqrt{3}k)}$

2. Tangente (Symmetrie Parabel) : $\underline{\alpha_2 = \pi - \alpha}$

d) $\frac{f(x) - f(x_a)}{x - x_a} = f'(x)$

$\frac{3(x-k)^2 - k^2 - (-k^2 - 3)}{x - k} = 6x - 6k$

$\underline{x = k \pm 1}$

$\underline{y = 3 - k^2}$

$f'(k \pm 1) = \pm 6 = \tan\beta_{1/2}$ $\underline{\beta_{12} = \pm\arctan 6}$

e) $\int_{x_1}^{x_2} f(x)\,dx = -12\sqrt{3}$

$[x^3 - 3kx^2 + 2k^2x]_{x_1}^{x_2} = -12\sqrt{3}$

$-\frac{4}{9}\sqrt{3}k^3 = -12\sqrt{3}$

$\underline{k = 3}$

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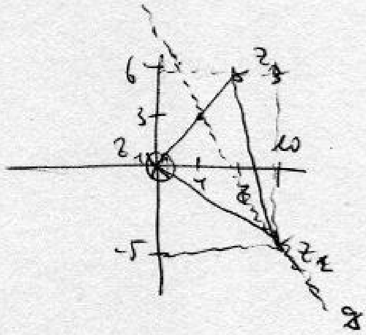
2. a)
$$10r + 8s = 6$$
$$-5r + 6s = 2$$

$$r = \frac{1}{5} \quad s = \frac{1}{2}$$



b) $A = 11 \cdot 10 - \frac{1}{2}(6 \cdot 8 + 6 \cdot 11 + 5 \cdot 10) = 50$

c) $z^2 = 8 + 6i \qquad |z_2| = \sqrt{8^2 + 6^2} = 10$

$(a + bi)^2 = 8 + 6i$

I $a^2 - b^2 = 8$

II $2ab = 6$

$a_1 = -3 \quad b_1 = -1 \quad : \quad z_1 = -3 - i$

$a_2 = 3 \quad b_2 = 1 \quad : \quad z_2 = 3 + i$

d) $z_M = \frac{z_2 + z_3}{2} = 4 + 3i$

$$\left.\begin{array}{l} z_1 z_3 : \begin{pmatrix} 8 \\ 6 \end{pmatrix} \\ z_M z_2 : \begin{pmatrix} 6 \\ -8 \end{pmatrix} \end{array}\right\} \boxed{\cdot}$$

e) $w = f(z) = (2 - i)z + 1$

$w_2 = f(z_2) = (2 - i)(10 - 5i) + 1 = 16 - 20i$

$w_3 = f(z_3) = (2 - i)(8 + 6i) + 1 = 23 + 4i$

$|2 - i| = \sqrt{5}$

$\varphi = -26{,}56°$

f: Drehung um $-26{,}6°$ (im Uhrzeigersinn)
Streckung um $\sqrt{5}$ } Zentrum Z ?
Verschiebung um 1 in Richtung der reellen Achse

Z: $f(z) = z$

$(2 - i)z + 1 = z$

$(1 - i)z = -1$

$z = -\frac{1}{1-i} = \frac{-1(1+i)}{1+1} = -\frac{1}{2} - \frac{1}{2}i$

$Z(-\frac{1}{2} | -\frac{1}{2})$

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3\. a) $\alpha: 2x + y + 2z = 6$

$U(3|0|0)\ ;\ V(0|6|0)\ ;\ W(0|0|3)$

$\underline{V_{OUVW}} = \frac{1}{6} \cdot 3 \cdot 6 \cdot 3 = \underline{9}$

b) HNF(α) $\quad \frac{2x + y + 2z - 6}{3} = 0$

$P(15|3|9) \quad \left| \frac{2 \cdot 15 + 3 + 2 \cdot 9 - 6}{3} \right| = \underline{15 = d(P;\alpha)}$

c) $g_{PQ}: \vec{x} = \begin{pmatrix} 15 \\ 3 \\ 9 \end{pmatrix} + s \begin{pmatrix} 42 \\ -15 \\ 33 \end{pmatrix} = \begin{pmatrix} 15 \\ 3 \\ 9 \end{pmatrix} + t \begin{pmatrix} 14 \\ -5 \\ 11 \end{pmatrix}$

in α: $\quad 2 \cdot (15 + 14t) + 3 - 5t + 2(9 + 11t) = 6$

$t = \frac{45}{11}$

$\underline{D\left(\frac{795}{11} \middle| -\frac{192}{11} \middle| 54\right)}$

d) $g_{PR}: \vec{x} = \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix} + t \begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \\ 7 \end{pmatrix} + s \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

$\beta \perp g: \quad 4x - 2y + z = 0$ durch 0

$\beta \cap g: \quad 4(7 + 4s) - 2(7 - 2s) + 7 + s = 0$

$s = 1$

$\underline{S(11|5|8)}$

e) n: Schnitt α mit $z = 0$: $n = (UV)$ $\quad \vec{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ -6 \\ 0 \end{pmatrix}$

α: $\quad ax + by + cz = d$

$W(0|0|3) \quad \underline{3c = d}$

$\downarrow$

$\vec{n}$ von α ~ $\begin{pmatrix} a \\ -2a \\ 0 \end{pmatrix}$

$\downarrow$

$ax - 2ay = 0$

$\underline{\alpha: 1x - 2y = 0}$

4. a) $4! = 24$

Bei 3 richtigen (von 4) muss der vierte auch richtig sein.

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b) $P(4) = \frac{1}{24}$

$P(3) = 0$

$P(2) = \frac{6}{24}$

$P(1) \frac{8}{24}$

$P(0) = \frac{9}{24}$

c) $E(X) = 1$

$\sigma(X) = 1$

d) $P(Y=k) = \frac{1}{24} + \frac{23}{24} \cdot \frac{1}{24} + \left(\frac{23}{24}\right)^2 \frac{1}{24} + \dots$

$$= \sum_{k=0}^{n} \frac{1}{24} \left(\frac{23}{24}\right)^k = \frac{1}{24} \sum_{k=0}^{n} \left(\frac{23}{24}\right)^k$$

$$= \frac{1}{24} \frac{1-\left(\frac{23}{24}\right)^{k+1}}{\underbrace{1-\frac{23}{24}}_{\frac{1}{24}}} = 1-\left(\frac{23}{24}\right)^{k+1}$$

$$\lim_{k \to \infty} \left(1-\left(\frac{23}{24}\right)^{k+1}\right) = 1$$

e) $E(Y) = 1 \cdot \frac{1}{24} + 2 \cdot \frac{23}{24} \frac{1}{24} + 3\left(\frac{23}{24}\right)^2 \cdot \frac{1}{24} + \dots$

$$= \frac{1}{24} \left(1 + 2 \cdot \frac{23}{24} + 3 \cdot \left(\frac{23}{24}\right)^2 + \dots\right)$$

$$= \frac{1}{24} \frac{1}{\left(1-\frac{23}{24}\right)^2} = 24$$

5 a) $\vec{c} \sim \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ d.h. $c = k \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

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$\vec{c} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 60 \quad = 6k + 6k$

$k = 5$

$\underline{\vec{c} = \begin{pmatrix} 15 \\ 10 \end{pmatrix}}$

b) $3^{2x} \cdot 3 + 3^x \cdot 3^{1,5} = 4$

$3u^2 + 3^{1,5} u - 4 = 0$

$u_1 = -\frac{4}{3}\sqrt{3} \qquad u_2 = \frac{1}{3}\sqrt{3}$

$3^x = \quad < 0 \qquad 3^x = 3^{-1/2}$

$\lightning$

$\underline{x = -1/2}$

c) $\binom{n+1}{n-1} \cdot \frac{2n!}{(n+2)!} = \frac{(n+1)!}{(n-1)!\,(2)!} \cdot \frac{2n(n-1)!}{(n+2)(n+1)!} = \underline{\frac{n}{n+2}}$

d) $\int \underset{v}{x} \cdot \underset{u'}{\sin x}\, dx = -x\cos x + \int \cos x\, dx$

$\underline{= -x\cos x + \sin x}$

e) $A\vec{x} = \lambda \vec{x}$

$\begin{vmatrix} 1,4 - \lambda & 0,8 \\ -0,25 & 0,5 - \lambda \end{vmatrix} = 0$

$\underline{\lambda_1 = 1}$

$\underline{\lambda_2 = \frac{9}{10}}$

$0,4u + 0,8v = 0 \qquad 0,5u + 0,8v = 0$

$u = 2v \qquad u = -\frac{8}{5}v$

$\underline{\vec{x}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}} \qquad \underline{\vec{x}_2 = \begin{pmatrix} -1,6 \\ 1 \end{pmatrix}}$