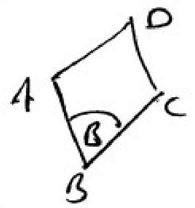
Mater EN H11




1.1.a)

$$\vec{AB} = \begin{pmatrix}-4\\-4\\-2\end{pmatrix}, \quad \vec{DC} = \begin{pmatrix}-4\\-4\\-2\end{pmatrix} \Big\} \text{ id.} \Rightarrow \text{Parallelogramm}$$

$$\vec{BC} = \begin{pmatrix}4\\2\\-4\end{pmatrix} \qquad |\vec{BC}| = |\vec{AB}|$$

$$\Rightarrow \text{Rhombus}$$

$$\cos\beta = -\frac{\vec{AB}\cdot\vec{BC}}{AB\cdot BC} = +\frac{\begin{pmatrix}4\\4\\2\end{pmatrix}\begin{pmatrix}4\\2\\-4\end{pmatrix}}{6\cdot 6} = \frac{16+8-8}{36} = \frac{4}{9}$$

$$\underline{\beta = 63{,}61^\circ} = \underline{\delta}\ ; \quad \underline{\alpha = \gamma} = 180^\circ - \beta = \underline{116{,}4^\circ}$$

b)

$$A_{ABCD} = |\vec{AB} \times \vec{BC}| = \left|\begin{pmatrix}4\\4\\2\end{pmatrix} \times \begin{pmatrix}4\\2\\-4\end{pmatrix}\right| = 4\left|\begin{pmatrix}2\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\1\\-2\end{pmatrix}\right| = 4\left|\begin{pmatrix}-5\\+6\\-2\end{pmatrix}\right|$$

$$= 4\sqrt{65}$$

$\cdot 3$

$$V = \frac{1}{3} G \cdot h \Rightarrow h = \frac{3V}{G} = \frac{3\cdot 1040}{4\sqrt{65}} = 12\cdot\sqrt{65}$$

Fuß F (Mitte AC): $\vec{r}_F = \frac{1}{2}\left(\begin{pmatrix}2\\4\\6\end{pmatrix} + \begin{pmatrix}2\\2\\0\end{pmatrix}\right) = \begin{pmatrix}2\\3\\3\end{pmatrix}$

$$\vec{FS} = 3\cdot 4\cdot\begin{pmatrix}-5\\6\\-2\end{pmatrix} = 12\cdot\begin{pmatrix}-5\\6\\-2\end{pmatrix}$$

$$\underline{\vec{r}_S} = \begin{pmatrix}2\\3\\3\end{pmatrix} + 12\cdot\begin{pmatrix}-5\\6\\-2\end{pmatrix} = \underline{\underline{\begin{pmatrix}-58\\75\\-21\end{pmatrix}}} \qquad S(-58|75|-21)$$

1.2

a) $x^2+4x+4+y^2-6y+9+z^2 = 12+4+9$

$(x+2)^2+(y-3)^2+z^2 = 25$

$M(-2|3|0) \qquad R=5$

b) $(-2+2)^2+(-2+4t-3)^2+(10-3t)^2=25$

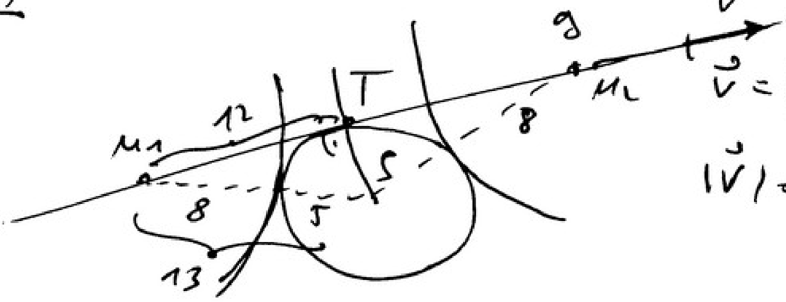
$t^2-4t+4=0$

$(t-2)^2=0$

$\underline{t=2}$ doppelte Lösung $\Rightarrow$ Berührung

$t$ in $g$: $\underline{T(-2|6|4)}$

c)




$\vec{v}=\begin{pmatrix}0\\4\\-3\end{pmatrix}$

$|\vec{v}|=5$

$\vec{r}_{M_{1/2}}=\vec{r}_T \pm \frac{12}{5}\cdot\vec{v}$

$\vec{r}_{M_1}=\begin{pmatrix}-2\\6\\4\end{pmatrix}+\frac{12}{5}\begin{pmatrix}0\\4\\-3\end{pmatrix}=\begin{pmatrix}-2\\78/5\\-16/5\end{pmatrix} \qquad \underline{M_1\left(-2\middle|\frac{78}{5}\middle|-\frac{16}{5}\right)}$

$\vec{r}_{M_2}=\begin{pmatrix}-2\\6\\4\end{pmatrix}-\frac{12}{5}\begin{pmatrix}0\\4\\-3\end{pmatrix}=\begin{pmatrix}-2\\-18/5\\56/5\end{pmatrix} \qquad \underline{M_2\left(-2\middle|-\frac{18}{5}\middle|\frac{56}{5}\right)}$

2.

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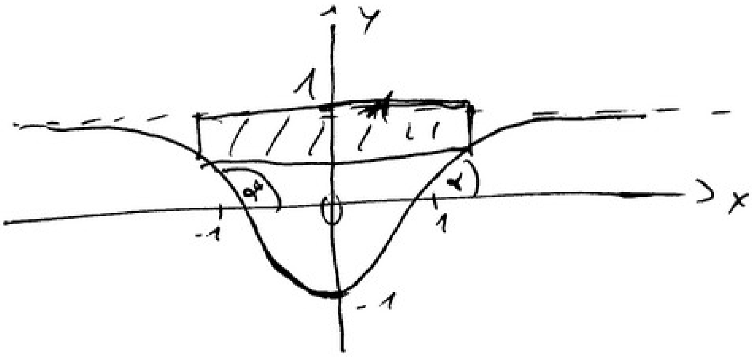


a)

$f(-x) = f(x)$ : A.S.

$D = \mathbb{R}$

NST: $e^{-x^2} = \frac{1}{2}$

$-x^2 = \ln \frac{1}{2}$

$\underline{x = \pm\sqrt{\ln 2}}$ $(= \pm\, 0{,}832)$

$f'(x) = 4x\, e^{-x^2}$

$f''(x) = 4e^{-x^2} - 8x^2 e^{-x^2} = 4(1-2x^2)\,e^{-x^2}$

$f'(x) = 0$

$\underline{x = 0}$ $\underline{y = -1}$ $f''(0) > 0 \Rightarrow Min(0|-1)$

$f''(x) = 0$

$x = \pm\sqrt{\tfrac{1}{2}} = \pm \tfrac{1}{2}\sqrt{2}$ $y = 1 - e^{-\frac{1}{2}}$

einfache NST -> VZW -> $\underline{WP(\pm \tfrac{1}{2}\sqrt{2} \mid 1 - e^{-\frac{1}{2}})}$

$\lim\limits_{|x|\to\infty} f(x) = \lim\limits_{|x|\to\infty} (1 - \underbrace{2e^{-x^2}}_{\to 0,\ > 0}) = \underline{\underline{1}}$ waag. A. $\Big\}$ Graph immer unterhalb $y = 1$

$\overset{\Rightarrow}{(\text{Extr.})}$ $W = [-1; 1[$

b) $\tan\alpha = f'(\sqrt{\ln 2}) = 4\cdot\sqrt{\ln 2}\,\frac{1}{e^{\ln 2}} = 2\sqrt{\ln 2}$

$\underline{\underline{\alpha = 59{,}01°}}$

$(\underline{\alpha^* = 120{,}99°})$

c) A.S: $b = 0$

$y = ax^2 + c$

$Min(0|-1)$ : $c = -1$

$y = ax^2 - 1$

NST $(\sqrt{\ln 2}\,|\,0)$ : $0 = a\cdot\ln 2 - 1$

$a = \frac{1}{\ln 2}$

$\underline{p: y = \frac{1}{\ln 2}\cdot x^2 - 1}$

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d) $A = x \cdot (1 - f(x))$

$A(x) = x \cdot 2 \cdot e^{-x^2} = \frac{1}{2} f'(x)$

$A'(x) = \frac{1}{2} f''(x) = 2(1-2x^2)\, e^{-x^2}$

$A'(x) = 0$

$\underline{x = \pm \frac{1}{2}\sqrt{2}}$ einfach, Extrema

$\underline{A(\frac{1}{2}\sqrt{2}) = \sqrt{2} \cdot e^{-\frac{1}{2}}}$

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3a) Bei jedem Griff gibt es zwei Möglichkeiten,
also $2^8 = 256$ insgesamt.

b) $P(K) = \frac{1}{2}$

$P(ZK) = \frac{1}{4}$ $\qquad E = \frac{1}{2} \cdot 10 + \frac{1}{4} \cdot (+8) + \frac{1}{4}(-4)$

$P(ZZ) = \frac{1}{4}$ $\qquad \underline{E = \quad 6}$

c) $P(\overset{E_1}{\text{Wurf enthält K}}) = P(\text{mind. ein K in 3 Würfen})$

$= 1 - P(\text{kein K in 3 Würfen}) = 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8}$

c₁) $P(\text{in 4 Würfen mind. ein } E_1) = 1 - P(\text{in 4 W. kein } E_1)$

$= 1 - \left(\frac{1}{8}\right)^4 = \frac{4095}{4096} = \underline{99{,}98\%}$

c₂) $P(\overset{E_2}{\text{Wurf ent. genau 2 K}}) = \binom{3}{2} \cdot \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = 3 \cdot \left(\frac{1}{2}\right)^3 = \frac{3}{8}$

$P(n \text{ Würfe mit mind. einmal } E_2) > 90\%$

$1 - P(n \text{ Würfe kein } E_2) > 0{,}9$

$\left(\frac{5}{8}\right)^n < 0{,}1$

$n > \log_{\frac{5}{8}} 0{,}1 = 4{,}899$

<u>Ab 5 Würfen</u>

d)

50% fair — 60K $\qquad P_1(60\text{K in } 100 \text{ W. } p=0{,}5) = \binom{100}{60} \cdot 0{,}5^{60}\, 0{,}5^{40}$

$= 1{,}084\%$

50% $\overline{\text{fair}}$ — 60K $\qquad P_2(60\text{K in } 100 \quad p=0{,}7) = \binom{100}{60}\, 0{,}7^{60}\, 0{,}3^{40}$

$= 0{,}849\%$

$P(\text{es war die } \overline{\text{faire}}) = \frac{0{,}5 \cdot P_2}{0{,}5 \cdot P_1 + 0{,}5 \cdot P_2} = \underline{\frac{P_2}{P_1 + P_2} = 43{,}9\%}$

4.1. $z^5 = 3-4i$

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$\varrho = \sqrt{3^2+4^2} = 5$

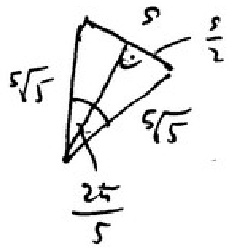
$\varphi = \arctan\left(-\frac{4}{3}\right) = 5{,}3559$

$z = \sqrt[5]{5} \cdot \operatorname{cis}\left(\frac{\varphi + 2k\pi}{5}\right)$ ; $k = \{0; 1; 2; 3; 4\}$

$\varrho = 1{,}38$ $\quad \varphi_0 = 1{,}071 \quad \varphi_2 = 3{,}58 \quad \varphi_4 = 6{,}098$

$\varphi_1 = 2{,}33 \quad \varphi_3 = 4{,}84$

5-Eck mit Umkreisradius $\sqrt[5]{5}$




$\frac{s}{2} = \sqrt[5]{5} \cdot \sin\left(\frac{2\pi}{10}\right)$

$s = 2 \cdot \sqrt[5]{5} \sin\left(\frac{2\pi}{10}\right)$

$\underline{\underline{s = 1{,}62}}$

4.2. a) $f(z) = (1+i)z + i = z$

$z + iz + i = z$

$\underline{z = -1}$

b) $az+b$ : Drehung (Zentrum 0) mit $\alpha$ (Arg. v. a) und Streckung um $|a|$

Verschiebung um b

$a = 1+i$ : $|a| = \sqrt{2}$ ; $\alpha = 45°$
$b = i$

Drehung um 45° und Streckung mit $\sqrt{2}$ sowie Verschiebung um 1 nach oben (y-Richtung, Img-Achse)

c) $f(z) = (1+i)z + i = w \quad | \; w = u+iv, \; z = x+iy$

$(1+i)(x+iy) + i = u+iv$

$x + iy + ix - y + i = u+iv$

$x - y + (x+y+1)i = u+iv$

$\left| \begin{array}{l} x-y = u \\ x+y+1 = v \end{array} \right|$

Hyp: $x \cdot y = 1$

$y = \frac{1}{x}$

$x - \frac{1}{x} = u$

$+ \quad x + \frac{1}{x} + 1 = v$

$2x + 1 = u+v$

$x = \frac{u+v-1}{2}$

$\frac{u+v-1}{2} - \frac{2}{u+v-1} = u$

$(u+v-1)^2 - 4 = 2u(u+v-1)$

$\underline{\underline{v = 1 \pm \sqrt{u^2+4}}}$

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4.3. a)

$$\begin{pmatrix} 1/3 & 0 \\ a & -1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ \boxed{3a-8} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \boxed{0} & 1 \end{pmatrix}$$

$$3a - 8 = 0$$

$$\underline{\underline{a = \frac{8}{3}}}$$

b)

$$\begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -y \end{pmatrix}$$ Spiegelung an x-Achse

c)

$$A \cdot e = \lambda e$$

$$(A - \lambda \mathbb{1})e = 0$$

$$|A - \lambda \mathbb{1}| = 0$$

$$\begin{vmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-1-\lambda) - 0 = 0$$

$$\underline{\lambda_1 = 3}$$

$$\underline{\lambda_2 = -1}$$

$\underline{\lambda_1 = 3:}$ $$\begin{pmatrix} 0 & 0 \\ 8 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 8x-4y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ $\underline{e_1 = \begin{pmatrix} x \\ 2x \end{pmatrix}}$

$$\underline{y = 2x}$$

$\underline{\lambda_2 = -1:}$ $$\begin{pmatrix} 4 & 0 \\ 8 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4x \\ 8x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ $\underline{e_2 = \begin{pmatrix} 0 \\ y \end{pmatrix}}$

$$\underline{x = 0}$$

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5.1.
$$\sum_{i=0}^{n} q^i = s_n \quad \Big| \cdot q$$

$$\sum_{i=0}^{n} q^{i+1} = s_n \cdot q$$

(−)

$$\sum_{i=0}^{n} q^i - \sum_{i=0}^{n} q^{i+1} = s_n - s_n q$$

$$\sum_{i=0}^{n} q^i - \sum_{i=1}^{n} q^i - q^{n+1} = s_n(1-q)$$

$$q^0 - q^{n+1} = s_n(1-q)$$

$$\underline{\underline{s_n}} = \frac{q^0 - q^{n+1}}{1-q} = \underline{\underline{\frac{1-q^{n+1}}{1-q}}}$$

5.2.
$$y = A \cdot \sin\left[a\left(x + \frac{b}{a}\right)\right] + B$$

$$W = \underbrace{[-1;3]}_{4} \;:\; 2A = 4 \qquad \underline{\underline{A = 2}} \qquad \underline{\underline{B = 1}}$$

Periode $4\pi$ : $\underline{\underline{a = \frac{1}{2}}}$

um $\pi$ nach rechts verschoben: $\frac{b}{a} = -\pi$

$$\underline{b = -\tfrac{1}{2}\pi}$$

$$y = 2 \cdot \sin\left(\tfrac{1}{2}x - \tfrac{1}{2}\pi\right) + 1 = 2$$

$$\sin\left(\tfrac{1}{2}x - \tfrac{\pi}{2}\right) = \tfrac{1}{2}$$

$$\tfrac{1}{2}x - \tfrac{\pi}{2} = \tfrac{\pi}{6} + z \cdot 2\pi \qquad \Big| \qquad \tfrac{1}{2}x - \tfrac{\pi}{2} = \tfrac{5}{6}\pi + z \cdot 2\pi$$

$$\underline{x_1 = \tfrac{4}{3}\pi + z \cdot 4\pi} \qquad \Big| \qquad \underline{x_2 = \tfrac{8}{3}\pi + z \cdot 4\pi}$$

5.3.
$$\int_0^2 x e^{2x}\,dx = \left[x \cdot \tfrac{1}{2}e^{2x}\right]_0^2 - \int_0^2 \tfrac{1}{2}e^{2x}\,dx$$

$$= e^4 - \left[\tfrac{1}{4}e^{2x}\right]_0^2$$

$$= e^4 - \tfrac{1}{4}e^4 + \tfrac{1}{4} = \underline{\underline{\tfrac{3}{4}e^4 + \tfrac{1}{4}}}$$