EN F17

1. $f(x) = ax^3 + bx^2 + cx + d$

a) $f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b \quad = 0$: lin. Gleichung, eine (genau) Lösung, VzW, also WP

b) $f' = 0$: 2 Lösungen, $D > 0$

$\underline{4b^2 - 12ac > 0}$

c) $f(x) = -\frac{1}{4}x^3 + x = 0$

$x = 0, \ x = \pm 2$

$\underline{A} = \int_0^2 f(x)\,dx = \left[-\frac{1}{16}x^4 + \frac{1}{2}x^2\right]_0^2 = \underline{1}$

d) $-\frac{1}{4}x^3 + x = mx$

$x\left(-\frac{1}{4}x^2 + 1 - m\right) = 0$

$x_1 = 2\sqrt{1-m}$

e) $\frac{1}{2} = \int_0^{x_1} (f - mx)\,dx =$

$\frac{1}{2} = \left[-\frac{1}{16}x^4 + \frac{1}{2}(1-m)x^2\right]_0^{2\sqrt{1-m}}$

$\frac{1}{2} = (m-1)^2$

$m = 1 \pm \frac{1}{2}\sqrt{2} \quad (+) \notin D$

$\underline{m = 1 - \frac{1}{2}\sqrt{2}}$

EN F11

2.

a) $z^2 - \bar{z}^2 = e^{2i\varphi} - e^{-2i\varphi}$

$= \cos 2\varphi + i \sin 2\varphi - (\overbrace{\cos(-2\varphi)}^{\cos 2\varphi} + i \overbrace{\sin(-2\varphi)}^{-\sin 2\varphi})$

$= \cos 2\varphi + i \sin 2\varphi - \cos 2\varphi + i \sin 2\varphi$

$= \underline{2i \sin 2\varphi \in \mathbb{I}}$

b) $\frac{e^{i\varphi} - e^{-i\varphi}}{2i} = \frac{2i \sin\varphi}{2i} = \underline{\sin\varphi}$

c) $z^3 = 27 \operatorname{cis}(216°)$

$\underline{z = 3 \operatorname{cis}(72° + 120° \cdot k)}$ $k = 0; 1; 2$

d) $1 + \frac{1}{10} i$ $\varphi = \arctan\left(\frac{1}{10}\right) = 5{,}71°$

$r = \sqrt{1{,}01}$

$(\sqrt{1{,}01}\, e^{i\varphi})^{10} = 1{,}01^5\, e^{i10\varphi} = 1{,}05101\,(0{,}54317 + i\,0{,}83962)$

$= 0{,}570877 \quad i \cdot 0{,}882448$

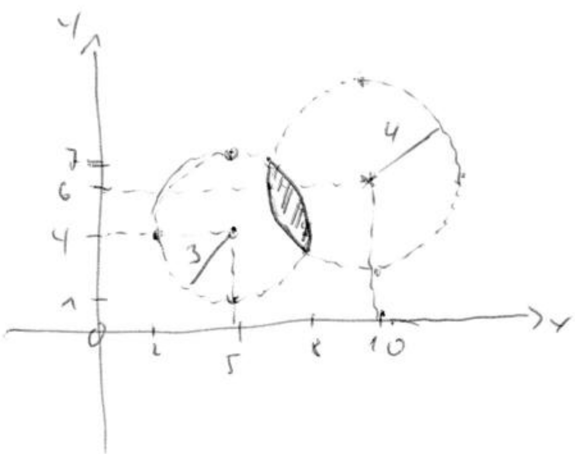
$- \quad e^i = 0{,}540302 + i\,0{,}84147$

$d \qquad 0{,}030575 - 0{,}040977\,i$

$r = 0{,}05113$

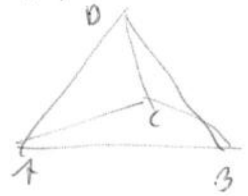
$\frac{r}{|e^i|} = \frac{0{,}05113}{1} = 5{,}113\,\%$

e) $|z - (5+4i)| \leq 3 \;\wedge\; |z - (10+6i)| \leq 4$



ENT 11

3. $A(0|0|0)$ $B(-1|32|15)$ $C(27|11|20)$ $D(-4|3|35)$



a) $\overline{AB} = \begin{pmatrix}-1\\32\\15\end{pmatrix}$ $\overline{AC} = \begin{pmatrix}27\\11\\20\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}28\\-21\\5\end{pmatrix}$

$\cos\alpha = \dfrac{\overline{AB}\cdot\overline{AC}}{AB \cdot AC} = \dfrac{625}{\underbrace{25\sqrt{2}\cdot 25\sqrt{2}}_{\text{gleichschenklig}}} = \dfrac{1}{2} \Rightarrow \underline{\alpha = 60°}$ } gleichseitig alle Winkel $\underline{60°}$

b) $\overline{AD} = \begin{pmatrix}-4\\3\\35\end{pmatrix}$ $|\overline{AD}| = 25\sqrt{2}$

$\overline{CD} = \begin{pmatrix}-31\\-8\\15\end{pmatrix}$ $|\overline{CD}| = 25\sqrt{2}$ } s.o. ⇒ regulärer Tet.

$\overline{BD} = \begin{pmatrix}-3\\-29\\20\end{pmatrix}$ $|\overline{BD}| = 25\sqrt{2}$

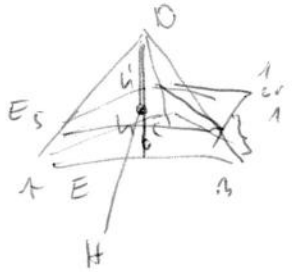
c) $\overline{AB} \times \overline{AC} = \begin{pmatrix}475\\425\\-875\end{pmatrix} \sim \underbrace{\begin{pmatrix}19\\17\\-35\end{pmatrix}}_{=\vec{n}}$: E_{ABC} $19x + 17y - 35z + d = 0$

$A(0|0|0)$: $d = 0$

$\underline{E_{ABC}: 19x + 17y - 35z = 0}$

d) $\vec{r}_S = \frac{1}{4}(\vec{r}_A + \vec{r}_B + \vec{r}_C + \vec{r}_D) = \frac{1}{2}\begin{pmatrix}11\\23\\35\end{pmatrix}$ $\underline{S\left(\frac{11}{2} \middle| \frac{23}{2} \middle| \frac{35}{2}\right)}$

e)



$\dfrac{V_0}{V} = \dfrac{1}{2} = \dfrac{\frac{1}{3}A'h'}{\frac{1}{3}Ah}$ $A = k^2 \cdot A'$, $h = k \cdot h'$

$\frac{1}{2} = \frac{1}{k^3} \Rightarrow k^3 = 2 \Rightarrow k = \sqrt[3]{2} \Rightarrow h' = \frac{1}{\sqrt[3]{2}} h$

$h = d(D;E)$ $\dfrac{19x + 17y - 35z}{25\sqrt{3}} = 0$

$D(-4|3|35)$: $\left|\dfrac{-1250}{25\sqrt{3}}\right| = \dfrac{50}{3}\sqrt{3} = h$

$h' = 22{,}912$

Abstand $(E, E') = h - h'$
$= 5{,}955$ $\dfrac{19x + 17y - 35z}{25\sqrt{3}} = 5{,}955$

E_S: $19x + 17y - 35z = 257{,}87$

$\overrightarrow{DH} = -\frac{1}{\sqrt[3]{2}} \cdot \vec{n} = -\frac{1}{\sqrt[3]{2}}\begin{pmatrix}19\\17\\-35\end{pmatrix}$

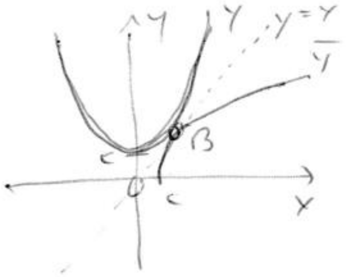
$\vec{r}_H = -\frac{1}{\sqrt[3]{2}}\vec{n} + \vec{r}_D = \begin{pmatrix}11{,}1\\16{,}5\\7{,}2\end{pmatrix}$ $\underline{H(11{,}1|16{,}5|7{,}2)}$

ENF12

5, a) $y = x^2 + c$

$\bar{x} = \bar{y}^2 + c$

$\bar{y} = \sqrt{\bar{x} - c}$



$\left.\begin{array}{ll} \text{I} & x^2 + c = x \\ \text{II} & 2x = 1 \end{array}\right\} \underline{c = \frac{1}{4}}$ $\quad \underline{B(\tfrac{1}{2} | \tfrac{1}{2})}$

b) $\begin{pmatrix} \cos 24° & -\sin 24° \\ +\sin 24° & \cos 24° \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 4\cos 24° - 3\sin 24° \\ +4\sin 24° + 3\cos 24° \end{pmatrix} = \begin{pmatrix} 2{,}43 \\ 4{,}36 \end{pmatrix}$ $\quad \underline{P^*(2{,}43 / 4{,}36)}$

c) $f(x) = x^{3/2}(2-x)$

$\underline{V} = \pi \int_0^2 x^3 (2-x)^2 \, dx = \pi \left[\frac{1}{6}x^6 - \frac{4}{5}x^5 + x^4 \right]_0^2 = \underline{\frac{16}{15}\pi}$

d) $AB = 0{,}5 \qquad BC = AB \cdot \cos 30° = \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} = \frac{1}{4}\sqrt{3}$

$CD = BC \cdot \cos 30° = \frac{1}{2} \cdot (\frac{1}{2}\sqrt{3})^2$

$DE = CD \cdot \cos 30° = \frac{1}{2}(\frac{1}{2}\sqrt{3})^3$

$\vdots$

$\underline{s} = \frac{1}{2}\left(1 + \frac{1}{2}\sqrt{3} + (\frac{1}{2}\sqrt{3})^2 + (\frac{1}{2}\sqrt{3})^3 + \dots\right)$

$= \frac{1}{2} \sum_{k=0}^{\infty} (\frac{1}{2}\sqrt{3})^k = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2}\sqrt{3}} = \underline{2 + \sqrt{3}}$

$\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$ für $|q| < 1$

$s = 2 = \frac{1}{2} \cdot \frac{1}{1-q}$

$q = \frac{3}{4}$

$\cos\alpha = \frac{3}{4}$

$\underline{\alpha = 41{,}41°}$

e) Anfang: $n = 1 \quad 4 \cdot 1^3 - 1 = 3$ teilbar 3

Annahme: $4n^3 - n \mid 3$

Schritt: $4(n+1)^3 - (n+1)$

$= 4n^3 + 12n^2 + 11n + 3$

$= \underbrace{\underbrace{4n^3 - n}_{\substack{|3 \\ \text{Annahme}}} + \underbrace{12n^2 + 12n + 3}_{\substack{3(4n^2+4n+1) \\ |3}}}_{|3}$

Also Induktionsschritt bewiesen,

somit $4n^3 - n$ teilbar 3 für alle $n \in \mathbb{N}$

EN F17

4. $A: \; p(1)=0 \quad p(2)=p(3)=p(5)=\frac{1}{6} \quad p(6)=\frac{2}{6}$

$B: \; p(1)=0=p(5) \quad p(2)=\frac{1}{6} \quad p(4)=\frac{2}{6} \quad p(6)=\frac{3}{6}$

a) ~~man~~ >10: $\left.\begin{array}{ll} 6+6 & : \left(\frac{2}{6}\right)^2 \\ 6+5 & : 2\cdot\frac{1}{6}\cdot\frac{2}{6} \end{array}\right\} \underline{\frac{2}{9} = 22{,}2\,\%}$

b) $P(\text{mind. ein } 6, A) > 0{,}99$

$1 - P(\text{nie } 6) > 0.99$

$\left(\frac{4}{6}\right)^n < 0{,}01$

$n > \log_{\frac{4}{6}} 0{,}01 = 11{,}35$

<u>Ab 12 Würfen</u>

c) $P(4\times\text{'4' in } 12, B) = \binom{12}{4}\cdot\left(\frac{2}{6}\right)^4\left(\frac{4}{6}\right)^8 = \underline{23{,}8\,\%}$

d) $P(\text{mind. } 2\times\text{'4' in } 12\,W.\ B)$

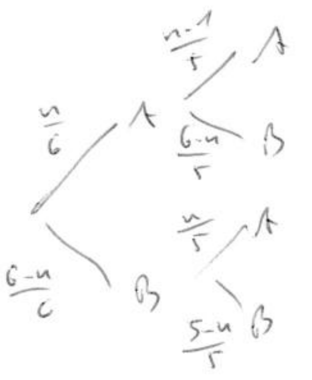
$= 1 - P(0\times\text{'4'} \vee 1\times\text{'4'}) = 1 - P(0\times\text{'4'}) - P(1\times\text{'4'})$

$= 1 - \left(\frac{4}{6}\right)^{12} - 12\cdot\left(\frac{2}{6}\right)^1\left(\frac{4}{6}\right)^{11} = \underline{94{,}6\,\%}$

e) $n: A$

$6-n: B$

Annahme: man nimmt zwei Würfel aus der Schachtel und würfelt '66'



$$\left.\begin{array}{ll} AA: & \frac{n(n-1)}{30} \cdot \left(\frac{2}{6}\right)^2 \\ AB: & 2\cdot\frac{n(6-n)}{30} \cdot \frac{2}{6}\cdot\frac{3}{6} \\ BB: & \frac{(6-n)(5-n)}{30} \cdot \left(\frac{3}{6}\right)^2 \end{array}\right\} \frac{n^2+31n+270}{1080} = P_{66}$$

$$P_{66} = \frac{2}{9}$$

$$\frac{n^2+31n+270}{1080} = \frac{2}{9}$$

$$\underline{n_1 = 1}$$

$$(n_2 = 30)$$