1. $f_k(x) = x^3 - 2kx^2 + k^2x \quad , k > 0 \quad ; \underline{1}\,WP$

a) NST : $x\,(x^2 - 2kx + k^2) = 0$

$x\,(x-k)^2 = 0$

$\underline{x = 0}$ einfach $\quad \underline{x = k}$ doppelt

b) $\underline{A} = \int_0^k f(x)\,dx = \left[\frac{1}{4}x^4 - \frac{2}{3}kx^3 + \frac{1}{2}k x^2\right]_0^k = \underline{\frac{1}{12}k^4}$

c) $f'(x) = 3x^2 - 4kx + k^2 = 0$

$\underline{x_1 = k} \qquad \underline{y_1 = 0}$ (dopp. NST)

$\underline{x_2 = \frac{1}{3}k} \qquad \underline{y_2 = \frac{4}{27}k^3}$

$f''(x) = 6x - 4k \qquad f''(k) = 2k > 0 \Rightarrow$ Min $(k|0)$

$f''(k/3) - 2k < 0 \Rightarrow$ Max $(\frac{1}{3}k \,|\, \frac{4}{27}k^3)$

d) $f''(x) = 0$

$x = \frac{2}{3}k \longrightarrow k = \frac{3}{2}x$

$y = \frac{2}{27}k^3 \longrightarrow y = \frac{2}{27}(\frac{3}{2}x)^3 =$

$C: \underline{y = \frac{1}{4}x^3}$ Ortskurve WP

e) $c = f \qquad c'(x) = \frac{3}{4}x^2$

$\frac{1}{4}x^3 = x^3 - 2kx^2 + k^2x$

$x_1 = 0 \qquad c'(0) = 0 \qquad f'(0) = k^2 \not\leftarrow \qquad c' \cdot f' = -1$ } nicht möglich

$x_2 = 2k \qquad c'(2k) = 3k^2 \qquad f'(2k) = 5k^2 \not\leftarrow$

$x_3 = \frac{2}{3}k \qquad c'(\frac{2}{3}k) = \frac{1}{3}k^2 \qquad f'(\frac{2}{3}k) = -\frac{1}{3}k^2 \leftarrow$ möglich

$\frac{1}{3}k^2 \cdot (-\frac{1}{3}k^2) = -1$

$k^4 = 9$

$\underline{k = (\pm)\sqrt{3}}$

LENT14

2. $z_1 = a + ib$; $z_2 = c + id$ $a - d \in \mathbb{R}$

a) $|z_1 - z_2|^2 + |z_1 + z_2|^2$

$= (z_1 - z_2)(z_1^* - z_2^*) + (z_1 + z_2)(z_1^* + z_2^*)$

$= 2 z_1 z_1^* + 2 z_2 z_2^*$

$= 2|z_1|^2 + 2|z_2|^2$ ✓

b) $z^3 = 6i = 6 \cdot \text{cis}\left(\frac{\pi}{2}\right)$

$\underline{z_k = \sqrt[3]{6} \cdot \text{cis}\left(\frac{\pi/2 + 2k\pi}{3}\right)}$; $k = 0, 1, 2$

c) $f(z) = (3+4i)z + 2 - 6i$

$f(z) = z = (3+4i)z + 2 - 6i$

$0 = (2+4i)z + 2 - 6i$

$\underline{z} = \frac{-2+6i}{2+4i} = \frac{(-2+6i)(2-4i)}{(2+4i)(2-4i)} = \frac{20+20i}{20} = \underline{1+i}$

d) $\underline{e^{3-2i}} = \frac{e^3}{(e^i)^2} = \left[\frac{e^3}{(\cos 1 + i \sin 1)^2} = \frac{e^3}{\cos^2 1 - \sin^2 1 + 2 \cos 1 \sin 1\, i}\right] = \frac{e^3}{e^{2i}}$

$= \frac{e^3}{\cos 2 + \sin 2\, i} = \frac{e^3(\cos 2 - \sin 2\, i)}{\cos^2 2 + \sin^2 2} = \cos 2 \cdot e^3 - i \sin 2 \cdot e^3$

$\underline{= -8{,}358 - i\, 18{,}26}$

e)

$a z^2 + b z + c = 0$

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. $b^2 - 4ac > 0$ d.h. $z \in \mathbb{R}$, also $z = \bar{z}$, somit $f(z) = 0 \Rightarrow f(\bar{z}) = 0$

2. " $= 0$ $z = -\frac{b}{2a}$, " " " "

3. $b^2 - 4ac < 0$ d.h. $z \in \mathbb{C}$, $z_1 = \frac{-b + i\sqrt{4ac - b^2}}{2a}$

$z_2 = \frac{-b - i\sqrt{4ac - b^2}}{2a}$

$\bar{z}_1 = z_2$, da z_1 Lsg. ist es auch $\bar{z}_1$ (und umgekehrt).

3. $A(1|5|2)$; $B(3|9|6)$; $C(7|5|8)$; $D(5|1|4)$ ENF14

a) $\vec{AB} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$ $\vec{DC} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$ $\vec{AB} = \vec{DC}$ und $\vec{BC} = \vec{AD}$ daher sind in einer Ebene da nur 2 lin. unabh. Vekt.

$\vec{BC} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$ $\vec{AD} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$

b) $\vec{AB} \times \vec{BC} = \begin{pmatrix} 24 \\ 12 \\ -24 \end{pmatrix} \sim \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \vec{n}$

$E: \quad 2x + y - 2z + d = 0 \quad (*)$

$A(1|5|2): \quad 2 + 5 - 4 + d = 0$

$\underline{E: \quad 2x + y - 2z - 3 = 0}$

c) $\vec{AX} = \begin{pmatrix} x-1 \\ -5 \\ -2 \end{pmatrix}$ $\vec{AX} \times \vec{AB} = \begin{pmatrix} -12 \\ -4x \\ 4x+6 \end{pmatrix}$

$A_{ABX} = \frac{1}{2} |\vec{u}| = \frac{1}{2}\sqrt{12^2 + (4x)^2 + (4x+6)^2} = 9$

$\Leftrightarrow 32x^2 + 48x + 180 = 162$

$\underline{\underline{x = -\frac{3}{4}}}$

d) $M = \left(\frac{1+7}{2} \middle| \frac{5+5}{2} \middle| \frac{2+8}{2}\right)$ Mitte(A, C)

$M(4|5|5)$

$\underline{g: \quad \vec{x} = \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix} + t \cdot \underbrace{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}}_{\vec{n}}}$

e) $a = AB = 6$ $A = 36$ $V = \frac{1}{3} A \cdot h$

$\underline{h} = \frac{3V}{A} = \frac{3 \cdot 180}{6} = \underline{90}$

$\vec{h} \sim \underbrace{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}}_{\text{Länge } 3} \longrightarrow \vec{h} = \pm 30 \cdot \vec{n}^0 = \pm \begin{pmatrix} 60 \\ 30 \\ -60 \end{pmatrix}$

$\vec{r}_{S_{1,2}} = \vec{r}_M \pm \vec{h} = \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix} \pm \begin{pmatrix} 60 \\ 30 \\ -60 \end{pmatrix}$

$\underline{S_1(64|35|-55)}$

$\underline{S_2(-56|-25|65)}$

ENF14

4. 4r, 6s; $p(r) = 0{,}4$

3mal Z. m. Z. $p(s) = 0{,}6$

a) P(Anzahl rot) $P(n;p;k) = \binom{3}{k} 0{,}4^k\, 0{,}6^{3-k}$

$k=0$: 21,6 %

$=1$: 43,2 %

$=2$: 28,8 %

$=3$: 6,4 %

b) $\underline{E} = \sum_{k=0}^{3} k \cdot p(k) \;\underline{= 1{,}2}$

$\sigma^2 = \sum_{k=0}^{3} (k-E)^2 p(k) = 0{,}76608$

$\underline{\sigma = 0{,}875}$

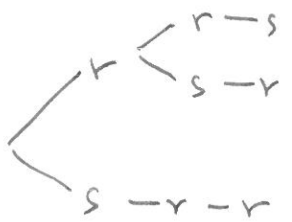
c) $E(3 \text{ rote Kegeln}) = 1 = n \cdot p$

$n = \frac{1}{p} = \frac{1}{0{,}064} = 15{,}6$

<u>16 mal</u>

d) $\sum_{k=0}^{3} k^2 \cdot p(k) = 2{,}16$ durchschnittlich Gewinn 2 fairer Einsatz

e)



$\underline{3 \cdot \frac{4 \cdot 3 \cdot 6}{10 \cdot 9 \cdot 8} = 30\,\%}$

LENF14

5 a) $\vec{c} = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ I. $a^2 + b^2 = 225$ $|15|$

II. $2a + 3b = 0$

$a = \frac{45}{13}\sqrt{13}$ $b = -\frac{30}{13}\sqrt{13}$

$\vec{c} = \frac{15\sqrt{13}}{13}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$

b) $y_C - y_B = 4 \rightarrow$ Amplitude : 2

$x_C - x_B = 6 \rightarrow$ Periode : 12

Mitte BC : $(7|4)$

$\frac{2\pi}{12} = \frac{\pi}{6}$

$y = 2 \cdot \sin\left(\frac{\pi}{6}(x-7)\right) + 4$

c) $\int_0^2 \sqrt{x}(2-x)\,dx = \int_0^2 \left(2\sqrt{x} - x^{3/2}\right)dx = \left[\frac{4}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^2$

$= \frac{16}{15}\sqrt{2}$

Mittelwert $= \frac{1}{2} \cdot \frac{16}{15}\sqrt{2} = \frac{8}{15}\sqrt{2}$

d) $y = x^3 + bx^2 + cx + d$

$y' = 3x^2 + 2bx + cx = 0$ d egal.

$D > 0$

$4b^2 - 12c > 0$

$b^2 > 3c$

e) Winkel gegenüber wie Seite, also γ:

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\gamma = \frac{1}{2}$

$\gamma = 60°$