EN F13

1. $f_k(x) = k^2 \sin(kx)$; $k>0$

a) $f_k(x) = 0$

$kx = z \cdot \pi$

$x = \frac{z \cdot \pi}{k}$ kleinste positive $z=1$ $x_0 = \frac{\pi}{k}$

$f'(x) = k^3 \cos(kx) = 0$

$kx = \frac{\pi}{2} + z \cdot \pi$

$x = \frac{\pi}{2k} + \frac{z\pi}{k}$ " $z=0$ $x = \frac{\pi}{2k}$ $y = k^2$

Hochpunkt weil $k^2 \cdot \sin(\,)$ (>0) ins positive startet.

$H\left(\frac{\pi}{2k} \middle| k^2\right)$

b) $x = \frac{\pi}{2k} \Rightarrow k = \frac{\pi}{2x}$

$y = k^2 = \frac{\pi^2}{4} \frac{1}{x^2}$

c) $\int_0^{x_0} f_k(x)\,dx = \left[-k \cos(kx)\right]_0^{\frac{\pi}{k}} = 2k$

d) $k=3$: $f_3(x) = 9 \cdot \sin(3x)$

$f_3'(x) = 27 \cdot \cos(3x)$

$P(\frac{\pi}{9}|..)$ $f_3'(\frac{\pi}{9}) = 27 \cdot \cos(\frac{\pi}{3}) = \frac{27}{2}$

$f_3(\frac{\pi}{9}) = 9 \cdot \sin(\frac{\pi}{3}) = \frac{9}{2}\sqrt{3}$

t: $y = \frac{27}{2}(x - \frac{\pi}{9}) + \frac{9}{2}\sqrt{3}$

Symmetrie: f ist P.S, da $\sin(ax)$ P.S. ist.

e) Monotoniebereich: von Min zu Max / Max z. Min (a) $: [-\frac{\pi}{2k}; \frac{\pi}{2k}] = D_k^*$

Umkehrfunktion: $x = k^2 \sin(ky)$

$\sin(ky) = \frac{x}{k^2}$

$ky = \arcsin(\frac{x}{k^2})$

$\bar{f}_k$: $y = \frac{1}{k} \arcsin(\frac{x}{k^2})$

ENF13

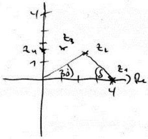
2. $z_1 = 4$; $z_2 = 4q$, $z_3 = 4q^2$

$q = \frac{\sqrt{2}}{2}\,\mathrm{cis}(30°)$
$= \frac{\sqrt{2}}{2}\left(\frac{1}{2}\sqrt{3} + i\,\frac{1}{2}\right)$
$q = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}i$

$q^2 = \frac{1}{2}\,\mathrm{cis}(60°)$
$q^2 = \frac{1}{4} + \frac{\sqrt{3}}{4}i$

$q^3 = \frac{\sqrt{2}}{4}\,\mathrm{cis}\,90°$
$q^3 = \frac{\sqrt{2}}{4}i$

a) $z_1 = 4$
$z_2 = \sqrt{6} + \sqrt{2}\cdot i$
$z_3 = 1 + \sqrt{3}i$
$z_4 = \sqrt{2}i$ Im



b) $S = \sum_{k=1}^{\infty} z_k = 4\cdot\frac{1}{1-q} = 4\cdot\frac{1}{\frac{4-\sqrt{6}-\sqrt{2}i}{4}} = 16\,\frac{4-\sqrt{6}+\sqrt{2}i}{(4-\sqrt{6})^2+2}$

$= 16\cdot\frac{4-\sqrt{6}+\sqrt{2}i}{24-8\sqrt{6}} = 2\,\frac{4-\sqrt{6}+\sqrt{2}i}{3-\sqrt{6}} = 2\,\frac{(4-\sqrt{6}+\sqrt{2}i)(3+\sqrt{6})}{9-6}$

$S = \frac{2}{3}(6+\sqrt{6}) + \frac{2}{3}(2\sqrt{3}+3\sqrt{2})i$

c) $d_1 = z_2 - z_1 = \sqrt{6} - 4 + \sqrt{2}i$
$|d_1| = \sqrt{(\sqrt{6}-4)^2 + 2} = \sqrt{24-8\sqrt{6}}$

d) $z_{k+1} - z_k = 4(q^k - q^{k-1}) = 4q^{k-1}(q-1) = d_k$
$|d_k| = |4q^{k-1}(q-1)| = 4\cdot|q^{k-1}||q-1|$
$= \underbrace{4\left|\left(\frac{\sqrt{2}}{2}\,\mathrm{cis}(30°)\right)^k\right|\left|\frac{\sqrt{6}}{4} - 1 + \frac{\sqrt{2}}{4}i\right|}_{d_1} = 4\cdot\left(\frac{\sqrt{2}}{2}\right)^k\cdot 1\cdot\sqrt{24-8\sqrt{6}}$

$L = \sum_{k=1}^{\infty} |d_k| = 2\sqrt{6-2\sqrt{6}}\cdot\frac{1}{1-\frac{\sqrt{2}}{2}} = 4(1+\sqrt{2})\sqrt{3-\sqrt{6}}$

e) $\tan\delta = \frac{\mathrm{Im}(z_2)}{\mathrm{Re}(z_1) - \mathrm{Re}(z_2)} = \frac{\sqrt{2}}{4-\sqrt{6}}$

$\delta = 42°$

3. ENF13

a) $\cos\alpha = \frac{\vec{AC}\cdot\vec{AB}}{AC \cdot AB} = \frac{\begin{pmatrix}-4\\0\\-2\end{pmatrix}\begin{pmatrix}-4\\2\\-4\end{pmatrix}}{|\cdot|\cdot|\cdot|} = \frac{24}{\sqrt{20}\sqrt{36}} = \frac{2}{5}\sqrt{5}$

$\underline{\alpha = 26{,}6 = 27°}$

b) $\vec{n} = \vec{AB} \times \vec{AC} = \begin{pmatrix}-4\\4\\8\end{pmatrix} \sim \begin{pmatrix}-1\\1\\2\end{pmatrix}$

$E_{ABC}: \quad -x + y + z + d = 0$

$B(0|3|0): \quad d = -3$

$\underline{E_{ABC}: \quad -x + y + z - 3 = 0}$

c) $P(2|4|3)$: HNF: $\frac{-x+y+z-3}{\sqrt{3}} = 0$

$d = \left|\frac{-2+4+3-3}{\sqrt{3}}\right| = \underline{\frac{2}{3}\sqrt{3}}$

d)

$\underline{d(A, g(PC))} = \frac{|\vec{PA} \times \vec{PC}|}{PC} = \frac{\left|\begin{pmatrix}2\\-3\\-1\end{pmatrix} \times \begin{pmatrix}-2\\-3\\-1\end{pmatrix}\right|}{\sqrt{14}} = \frac{\left|\begin{pmatrix}6\\0\\-12\end{pmatrix}\right|}{\sqrt{14}} = \underline{\frac{30}{7}\sqrt{14}}$

e) Q liegen auf der Mittelsenkrechtenebene $\hat{=}\ \Sigma$ zu AB mit $z = 0$

Mitte AB: $M(2|2|2)$

$\vec{n} = \vec{AB} = \begin{pmatrix}-4\\2\\-4\end{pmatrix} \sim \begin{pmatrix}2\\-1\\2\end{pmatrix}$

$\Sigma: \quad 2x - y + 2z + d = 0$

$M(2|2|2) \quad 4 - 2 + 4 + d = 0$

$d = -6$

$\Sigma: \quad 2x - y + 2z - 6 = 0$

Grundriss: $2x - y - 6 = 0$

$\underline{y = 2x - 6}$

LENF13

4\. a) Raten: $p = \frac{1}{3}$ $\quad q = \frac{2}{3}$

$\underline{E} = n \cdot p = \underline{\frac{1}{3}n}$

b) $Var = npq = n \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}n$

$\underline{\sigma = \frac{1}{3}\sqrt{2n}}$

c) $B(9; 3; \frac{1}{3}) = \binom{9}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^6 = 27{,}3\,\%$

d) $\sum_{k=4}^{7} B(7; k; \frac{1}{3}) = 17{,}3\,\%$

e) $\sum_{k=2}^{n} B(n; k; \frac{1}{3}) > 99\,\%$

$1 - \sum_{k=0}^{1} B(n; k; \frac{1}{3}) > 0{,}99$

$\left(\frac{2}{3}\right)^n + n \cdot \left(\frac{1}{3}\right) \cdot \left(\frac{2}{3}\right)^{n-1} < 0{,}01$

$\underline{n \geq 7}$

5, a) $N(t) = N_0 \cdot q^t = 2N_0$ $q = 1+p = 1{,}05$

$T = \log_q 2$

$\underline{T = 14{,}2 \text{ Jahre}}$

b) $y_{max} = 10$, $y_{min} = 2 \Rightarrow$ Amplitude 4

Periode: $24 \rightarrow \omega = \frac{24}{2\pi} = \frac{12}{\pi}$

$f(0) = 6$

$\underline{y(x) = 4 \cdot \sin(\frac{12}{\pi} x) + 6}$

c) $\lim_{x \to 0} \frac{1 - \cos^3 x}{\sin^2 x} = \lim_{x \to 0} \frac{1 - \cos^3 x}{1 - \cos^2 x} = \lim_{x \to 0} \frac{\cos^2 x + \cos x + 1}{\cos x + 1} = \underline{\frac{3}{2}}$

$(a^3 - 1) : (a^2 - 1) = a^2 + a + 1$

$a^3 - a^2$

$a^2 - 1$

$a^2 - a$

$a - 1$

d)

$3 \log_x 3 + \log_x 3x = 2$

$3 \log_x 3 + \log_x 3 + \log_x x = 2$

$4 \log_x 3 + 1 = 2$

$\log_x 3 = \frac{1}{4}$

$x^{\frac{1}{4}} = 3$

$\underline{x = 3^4 = 81}$

e) $\int_0^{\pi/4} x \cos(2x)\, dx = \left[x \cdot \frac{1}{2} \sin(2x)\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{2} \sin(2x)\, dx$

u v'

$= \frac{\pi}{8} \cdot 1 - 0 - \left[-\frac{1}{4} \cos(2x)\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{8} + \frac{1}{4}(0 - 1)$

$\underline{= \frac{\pi}{8} - \frac{1}{4}}$