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1. $f(x) = \ln x - (\ln x)^2$

a) $\underline{\ln x}\,(\underbrace{1 - \ln x}) = 0$

$\underline{x = 1}$ $\quad$ $\underline{x = e}$

$f'(x) = \frac{1}{x} - 2\ln x \cdot \frac{1}{x} = \frac{1}{x}(1 - 2\ln x)$

b) $\frac{1}{x}(\underbrace{1 - 2\ln x}) = 0$

$\underline{x = \sqrt{e}}$

$\underline{y = \frac{1}{4}}$ $\quad$ $\underline{A(\sqrt{e} \mid \frac{1}{4})}$

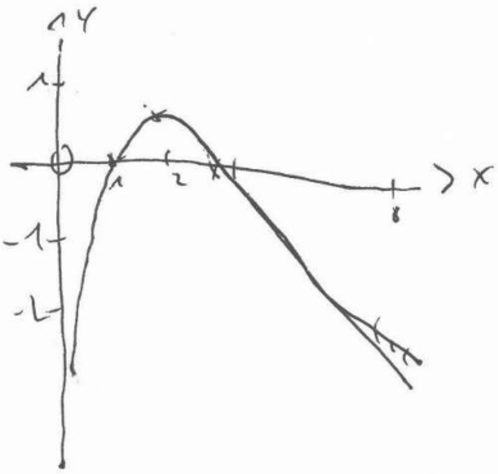
c) $f''(x) = -\frac{1}{x^2}(1 - 2\ln x) + \frac{1}{x}\left(-\frac{2}{x}\right)$

$= +\frac{1}{x^2}(-3 + 2\ln x) = 0$

$\underline{x = e^{\frac{3}{2}}}$

$\underline{y = -\frac{3}{4}}$ $\quad$ $\underline{B(\sqrt{e^3} \mid -\frac{3}{4})}$

d)



e) $F(x) = -3x + 3x\ln x - x(\ln x)^2$

$F'(x) = -3 + 3\ln x + 3 - (\ln x)^2 - 2\ln x$

$= \ln x - (\ln x)^2 = f(x)$ ✓

$$\int_1^e f(x)\,dx = \left[-3x + 3x\ln x - x(\ln x)^2\right]_1^e$$

$\underline{= 3 - e}$

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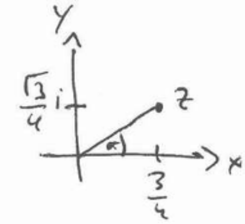
2. $w = f(z) = \frac{3+\sqrt{3}i}{4} \cdot z$

$z_0 = 4 \; ; \; z_{n+1} = f(z_n)$

a) $z = \frac{3+\sqrt{3}i}{4} = \frac{3}{4} + \frac{\sqrt{3}}{4} i$



$= \frac{1}{2}\sqrt{3}\, e^{\frac{\pi}{6} i}$

$= \underline{\underline{\frac{1}{2}\sqrt{3}\, \mathrm{cis}(30°)}}$

$\tan\alpha = \frac{\sqrt{3}}{3}$ $\qquad |z| = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2}$

$\alpha = 30°$ $\qquad = \frac{1}{2}\sqrt{3}$

Drehstreckung: Winkel 30°
Faktor $\frac{1}{2}\sqrt{3}$

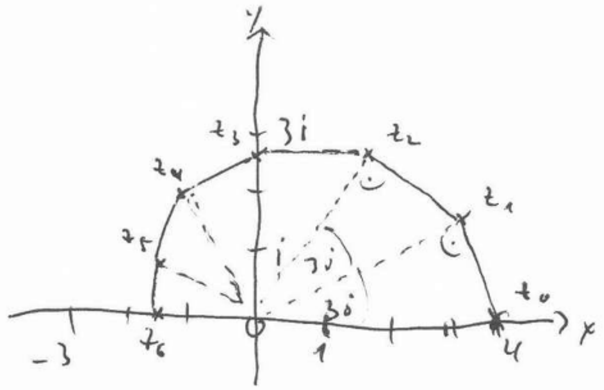
b) $z_n = \left(\frac{3+\sqrt{3}i}{4}\right)^n z_0 = \left(\frac{3+\sqrt{3}i}{4}\right)^n \cdot 4$

$z_2 = \left(\frac{3+\sqrt{3}i}{4}\right)^2 \cdot 4 = \frac{9 + 6\sqrt{3}i - 3}{16} 4 = \underline{\underline{\frac{3}{2} + \frac{3}{2}\sqrt{3} i}}$

c) $z_n = \left(\frac{3+\sqrt{3}i}{4}\right)^n \cdot 4 = \left(\frac{1}{2}\sqrt{3}\, \mathrm{cis}(30°)\right)^n \cdot 4 = 4 \cdot \left(\frac{\sqrt{3}}{2}\right)^n \cdot \mathrm{cis}(30° \cdot n)$

$z_0 = 4 \; ; \; z_1 = 2\sqrt{3}\, \mathrm{cis}(30°) \; ; \; z_2 = 3\, \mathrm{cis}(60°) \; ; \; z_3 = \frac{3}{2}\sqrt{3}\, \mathrm{cis}(90°)$

$z_4 = \frac{9}{4}\, \mathrm{cis}(120°) \; ; \; z_5 = \frac{9}{8}\sqrt{3}\, \mathrm{cis}(150°) \; ; \; z_6 = \frac{27}{16}\, \mathrm{cis}(180°)$



d) $\left|\frac{z_{n+1}}{z_n}\right| = \frac{\sqrt{3}}{2}$; Winkel 30°, also ist $\Delta\, 0 z_n z_{n+1}$ rechtwinklig

somit ~~...~~ $\overline{z_{n+1} z_n} = |z_n| \cdot \sin 30° = \frac{1}{2} |z_n|$

also $|z_n| = \left(\frac{1}{2}\right)^n \cdot 4$

$\underline{\underline{L}} = \lim_{n\to\infty} \sum_{i=0}^{n} |z_i| = \lim_{n\to\infty} \sum_{i=0}^{n} \left(\frac{1}{2}\right)^i \cdot 4 = \lim_{n\to\infty} \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} \cdot 4 = \underline{8}$

e) $\frac{z_m}{z_0} = \frac{z_1}{z_m} \Rightarrow z_m^2 = z_1 \cdot z_0 = 12 + 4\sqrt{3} i = 8\sqrt{3}\, \mathrm{cis}(30°)$

$Re(z_m) > 0$ $\qquad z_m = \sqrt{8\sqrt{3}} \cdot \mathrm{cis}\left(\frac{30° + 2k \cdot 180°}{2}\right) \quad k = 0,1$ $\qquad \underline{\underline{z_m = 2\sqrt{2\sqrt{3}}\, \mathrm{cis}(15°)}}$

$z_{m,0} = 2\sqrt{2\sqrt{3}} \cdot \mathrm{cis}(15°) \; ; \; z_{m,1} = 2\sqrt{2\sqrt{3}} \cdot \mathrm{cis}(195°)$

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3. $A(6|5|2)$ $B(6|6|7)$ $Q(2|17|12)$

a) $\underline{\underline{A_{\Delta ABQ}}} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AQ}| = \frac{1}{2}\left|\begin{pmatrix}0\\1\\5\end{pmatrix} \times \begin{pmatrix}-4\\12\\10\end{pmatrix}\right| = \frac{1}{2}\left|\begin{pmatrix}-50\\-20\\4\end{pmatrix}\right| = \underline{\underline{27}}$

b) $E_{ABQ}: \quad -50x - 20y + 4z + d = 0$

$\in A(6|5|2): \quad d = 192$

HNF $\quad \dfrac{-50x - 20y + 4z + 192}{54} = 0$

$(0|0|0): \quad \dfrac{192}{54} = \underline{\underline{\dfrac{32}{9}}}$

c) $X = (0|y|z)$: I $|\overrightarrow{AX}| = |\overrightarrow{BX}|$: $6^2 + (y-5)^2 + (z-2)^2 = 6^2 + (y-6)^2 + (z-7)^2$

II. $|\overrightarrow{AX}| = |\overrightarrow{CX}|$: " $= 9 + (y+3)^2 + (z-7)^2$

$y = 3 \quad z = 5$

$\underline{\underline{X(0|3|5)}}$ $\quad \underline{\underline{u = |\overrightarrow{AX}| = 7}}$

d) $g: \vec{X} = \underset{P}{\begin{pmatrix}15\\9\\6\end{pmatrix}} + t\underset{\vec{v}}{\begin{pmatrix}4\\3\\1\end{pmatrix}}$ $\quad \overrightarrow{AP} = \begin{pmatrix}9\\4\\4\end{pmatrix}$ $\quad \cos\alpha = \dfrac{\overrightarrow{AP}\cdot\vec{v}}{AP \cdot v} = \dfrac{52}{\sqrt{113}\cdot\sqrt{26}}$

$\alpha = 16{,}39°$

$\underline{d(A;g)} = AP \cdot \sin\alpha = \underline{\underline{3}}$

$\sin^2\alpha = 1 - \cos^2\alpha = \frac{9}{113}$; $\underline{\underline{d(A;g)}} = \sqrt{113} \cdot \frac{3}{\sqrt{113}} = \underline{\underline{3}}$

e) $K: \quad x^2 + y^2 + z^2 = 518 \quad \cap g$

$(4t+15)^2 + (3t+9)^2 + (t+6)^2 = 518$

$t_1 = \frac{11}{13}$

$t_2 = -8$

Alle Punkte mit $\underline{\underline{t \in]-\infty; -8[\cup]\tfrac{11}{13}; \infty[}}$

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4.

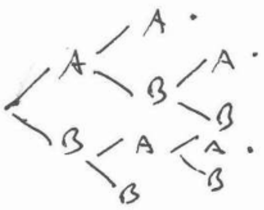
a) $P(\text{Anna einen Punkt in 3 Spielen}) = 3 \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^2 = \underline{\underline{\frac{4}{9} = 44{,}4\%}}$

b) $P(\text{Anna höchstens 4 Punkte in 12 Spielen})$

$$= \sum_{i=0}^{4} \binom{12}{i} \left(\frac{1}{3}\right)^i \left(\frac{2}{3}\right)^{12-i} = \left(\frac{2}{3}\right)^{12} + 12 \cdot \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{11} + \frac{12!}{2!\,10!}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{10} + \frac{12!}{3!\,9!}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^9 + \frac{12!}{4!\,8!}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^8$$

$$= \underline{\underline{63{,}2\%}}$$

c) $P(\text{beim, wer zuerst 2 Punkte}) = \left(\frac{1}{3}\right)^2 + 2 \cdot \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} = \underline{\underline{\frac{7}{27} = 25{,}9\%}}$



d) $P(\text{In } n \text{ Versuchen mind 2 Punkte}) > 0{,}99$

$1 - P(n \text{ 0 od. 1 Punkt}) > 0{,}99$

$1 - \left[\left(\frac{2}{3}\right)^n + n \cdot \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{n-1}\right] > 0{,}99$

$\left(\frac{2}{3}\right)^n + n\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{n-1} < 0{,}01$

$\underline{\underline{n \geq 17}}$

e) $\underline{\underline{E}} = 18 \cdot P(\text{Summe der zwei Augen} > 8) + 9 \overbrace{P(\text{Summe zwei Augen} \leq 8)}^{1 - P(>8)}$

$= 9 + 9\,P(>8) = \underline{\underline{11{,}5}}$

$P(>8)$: $\frac{1}{36} \cdot (1 + 2 + 3 + 4) = \frac{5}{18}$

Summe:	12	11	10	9
	6,6	6,5	6,4	6,3
		5,6	4,6	3,6
			5,5	5,4
				4,5

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5 a) $f(x) = \frac{5}{2}\sin\left(\frac{x}{2}\right) + \frac{1}{2}$

Streckung Y, Stauchung X, Verschiebung Y

b) $\sin x = \cos x$

$x = 45°$

$f'(x) = \cos x$

$g'(x) = -\sin x$

$f'(45°) = \frac{1}{2}\sqrt{2} = \tan\alpha \Rightarrow \alpha = 35{,}26°$

$g'(45°) = -\frac{1}{2}\sqrt{2} = \tan\beta \Rightarrow \beta = -35{,}26$

Schnitt : $70{,}5°$

$\approx 71°$

c) $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^n b^{n-i}$

$(1+1)^n = 2^n = \sum_{i=0}^{n} \binom{n}{i} 1^n 1^{n-i}$

$2^n = \sum_{i=0}^{n} \binom{n}{i} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$

d) $A(2|3|3)$ $B(-3|-2|2)$ $C(0|0|z)$

$\vec{AC} \cdot \vec{BC} = 0$

$\begin{pmatrix} -2 \\ -3 \\ z-3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ z-2 \end{pmatrix} = 0$

$-6 - 6 + (z-3)(z-2) = 0$

$z_1 = 6$

$z_2 = -1$

$C_1(0|0|6)$

$C_2(0|0|-1)$

e) $\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{e^{3x-\pi} - 1} = \lim_{x \to \frac{\pi}{3}} \frac{-\sin x}{3e^{3x-\pi}} = \frac{-\frac{1}{2}\sqrt{3}}{3} = -\frac{1}{6}\sqrt{3}$